Assignment 4(part 1) solutions

Question 1

KCL @ V_0: $(6 - V_0)/150 - V_0/600 + 0.02 = 0$

$$\Rightarrow V_0 = 7.2\text{ V}$$

$P_{6V} = 6\,I_V = 6\,[(6 - V_0)/150] = -48\text{ mW}$ (supplied!)

$P_{20mA} = 0.02\,V_I = 0.02\,(V_0 + 2) = 184\text{ mW}$ (supplied!)

Question 2

Negligible open-circuit losses mean that the battery can be accurately modeled by an independent voltage source (ideal emf source) of value V_{OC}, connected in series with a single internal resistance R_{INT}. Therefore, the object is to find the values of V_{OC} and R_{INT}, based on current measurements.

The exact short-circuit current of the battery, say I_{SC}, may be measured by connecting the ideal ammeter across the terminals of the battery, which provides: $V_{OC} = I_{SC} R_{INT}$.

The exact 10kΩ load current for the battery, say I_L, can be measured by connecting the battery, resistor and ammeter into a single series loop. This setup provides the relation: $V_{OC} = I_L (10{,}000 + R_{INT})$.

Combining these two V_{OC} relations yields the expression: $I_{SC} R_{INT} = I_L (10{,}000 + R_{INT})$, from which it is evident that the "measured" value of $R_{INT} = 10{,}000\, I_L / (I_{SC} - I_L)$.

Finally, substituting this result for R_{INT} in either of the two V_{OC} relations provided above yields an expression for the exact value of V_{OC}, e.g., $V_{OC} = 10{,}000\, I_{SC} I_L / (I_{SC} - I_L)$.

Question 3

(a) $I_S = 0.01V_0$; $V_S = -300\,(0.01V_0) - V_0 = \underline{-4V_0}$;

$\Rightarrow V_S = \underline{-400}\, I_S \quad \Rightarrow V_S \propto I_S \quad \Rightarrow$ Ohmic device.

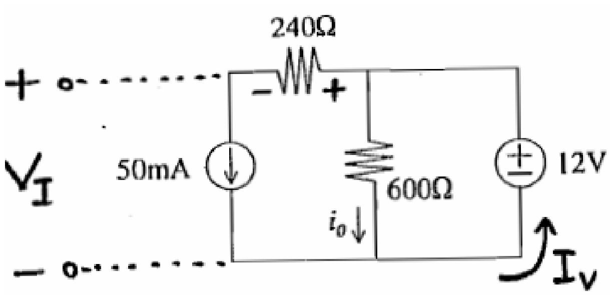
(b) $R_{EQ} = V_S / I_S = -400\,\Omega$

(c) $P_{Supplied} = I_S(-V_S) = (0.01V_0)(\underline{4V_0}) = 0.04\,V_0^2$;

KCL: $(V_0 - 5)/100 + V_0/200 - 0.01V_0 = 0$;

$\Rightarrow V_0 = 10V$; $\Rightarrow P_{Supplied} = 4W$

Question 4



4. (a) KVL: $V_I = 12 - 0.05 \times 240 = 0$;

$\Rightarrow P_I = 0.05\,V_I = 0\,W$

KCL: $I_V = 0.05 + 12/600 = 70\,mA$

$\Rightarrow P_V = 12\,I_V = 840\,mW$

(b) Model: ideal ammeter in series with 30Ω.

Connection: meter in series with 600Ω.

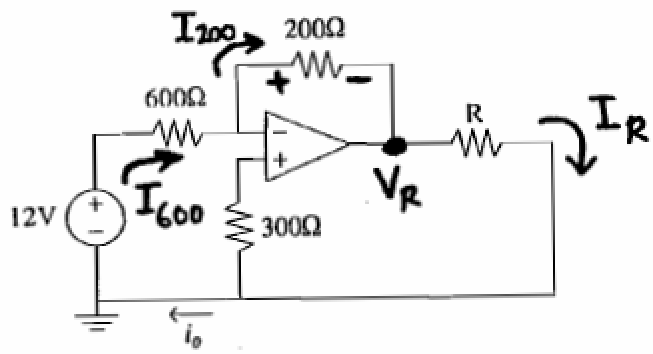
$\Rightarrow i_0 = 12/(600+30) \cong 19\,mA$

(c) KVL $\Rightarrow P_I = 0\,W$; (same as before)

KCL: $I_V \cong 0.05 + 0.019 = 69\,mA$

$\Rightarrow P_V \cong 12\,I_V = 828\,mW$

Question 5



(a) $i_0 = I_R = V_R / 4000$; $V_R = -200\, I_{200}$;

$I_{200} = I_{600} = 12 / 600 = 20\text{mA} \Rightarrow i_0 = -1\text{ mA}$

(b) $P_{heat} = 600\,(I_{600})^2 + 200\,(I_{200})^2 + (V_R)^2 / R$;

$P_{heat} = 0.32 + 16 / R$; $P_{12V} = 12\, I_{600} = 0.24\text{ W}$

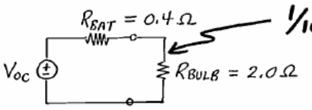
$\Rightarrow$ Solve $P_{12V} = 0.5\, P_{heat}$ for R

$0.24 = 0.16 + 8 / R \Rightarrow R = 100\,\Omega$

Question 6

(Extra space for solution to #5)

CIRCUIT MODEL: (BATTERY + BULB)



1/10 (CIRCUIT CONNECTION ONLY)

NB. BATTERY MODEL SHUNT RESISTOR IS NOT USED BECAUSE O/C BATTERY LOSSES ARE NEGLIGIBLE

1. OHMIC BULB RATED AT 72W FOR 12V (d.c.).

$$\Rightarrow R_{BULB} = \frac{V^2}{P} = \frac{144}{72} = 2.0\,\Omega \quad 2/10$$

2. $I_{BULB}(0+) = 5A \Rightarrow R_{BAT}$.

$$V_{BULB} = (5)(2) = 10V;$$

$$\Rightarrow V_{R_{BAT}} = V_{OC} - V_{BULB} = 12 - 10 = 2V;$$

(AT TIME 0+)

$$\Rightarrow R_{BAT} = \frac{V_{BAT}(0+)}{I_{BAT}(0+)} = \frac{2}{5} = 0.4\,\Omega \quad 2/10$$

(CON'D)

PROBLEM ANALYSIS / SOLUTION:

$$V_{BULB}(t>0) = \frac{2}{2.4} V_{OC}(t>0) \quad \ldots \text{ASSUMES ?}$$

$\Rightarrow V_{OC} = 12V \rightarrow V_{BULB} = 10V \quad (\geq 6V)$

$V_{OC} = 9V \rightarrow V_{BULB} = 7.5V \quad (\geq 6V)$

$V_{OC} = 6V \rightarrow V_{BULB} = 5V \quad (< 6V!)$

$\Rightarrow$ BULB FAILS WHEN V_{OC} SWITCHES FROM 9V DOWN TO 6V.

2/5 (2/10) $\Rightarrow$ 54 kJ REMAIN IN BATTERY AT TIME OF BULB FAILURE (AS PER V_{OC} PLOT).

TIME TO DISSIPATE 144 − 54 = 90 kJ:

FIRST 72 kJ AT 12V $\Rightarrow \frac{144}{2.4} = 60W \Rightarrow$ 20 MINS

\+

NEXT 18 kJ AT 9V $\Rightarrow \frac{81}{2.4} = 33.75W \Rightarrow$ 8 MINS 53 SEC

2/5 (2/10) $\Rightarrow$ TOTAL TIME TO FAILURE = 28 MINS 53 SEC

TOTAL ENERGY DISSIPATED BY BULB:

BY POWER DIVISION BETWEEN R_{BAT} AND R_{BULB}, THE FRACTION OF 90kJ DISSIPATED BY R_{BULB} IS

1/5 (1/10) $\frac{2}{2.4} \times 90kJ = 75kJ$.

Question 7

(a) KCL @ V_o : (ASSUME LOWER NODE AS GROUND)

$$2 - \frac{V_o}{200} - 3I_o = 0 \; ; \; V_o = 1000\, I_o$$

$$\Rightarrow V_o = 250\text{V} \; (2) \quad \text{AND} \quad I_o = 250\text{ mA} \; (2) \,//$$

(b) MEASURE V_o USING $R_m = 3000\Omega$:

$$2 - \frac{V_M}{3000} - \frac{V_M}{200} - 3I_o' = 0 \; ; \; V_M = 1000\, I_o'$$

$$\Rightarrow V_m = 240\text{V} \; (3) \,// \quad (R_m = 26\Omega \Rightarrow V_m \approx 43\text{V})$$

MEASURE I_o USING $R_m = 26\Omega$:

$$2 - \frac{V_o'}{200} - 3I_m = 0 \; ; \; V_o' = (1000+26)\, I_m$$

$$\Rightarrow I_M = 246\text{mA} \; (3) \,// \quad (R_m = 3000\Omega \Rightarrow I_m \approx 87\text{mA})$$

(c) SINCE I_o INDEPENDENT OF VALUE OF 300Ω, AND V_o INDEPENDENT OF VALUE OF 600Ω RESISTANCE, MEASURE I_{300} AND V_{600} (USING EITHER R_m !).

THEN: $I_m = \frac{1}{2} I_{300}$ (= I_o ACTUAL), AND

$V_M = \frac{1}{3} V_{600}$ (= V_o ACTUAL) //

Question 8

(a) KCL @ $-V_A$:

$$\frac{4+V_A}{1000} + \frac{V_A}{5000} + \frac{V_A - V_o}{2500} = 0$$

$$\Rightarrow 8V_A - 2V_o = -20$$

KCL @ $-V_o$:

$$\frac{V_o - V_A}{2500} + \frac{\alpha V_A + V_o}{1000} + \frac{V_o}{5000} = 0$$

$$\Rightarrow (2 - 5\alpha) V_A = 8 V_o$$

ELIMINATE V_o : $V_A = -16/(6+\alpha)$

FOR $\alpha \to \infty$: $V_A \to 0$; $V_o = 4V_A + 10 \to 10\text{V}$;

$$V_o/V_S = 10/4 = 2.5 \,//$$

(b) $P_{\diamond}^{SUP} = (\alpha V_A) I$

FOR $V_A = 0$: $I = -(I_S + I_o)$

$$= -\left(\frac{4}{1000} + \frac{10}{5000}\right) = -6mA \;;$$

BY KVL: ($V_A \to 0$) $\alpha V_A = -V_o + 1000 I = -16V$;

$$\Rightarrow P_{\diamond}^{SUP} = (-16)(-0.006) = +96 mW$$

(c) $P_{R_A} = V_A^2 / R_A$ · WHERE $V_A = -16/(6+\alpha)$;

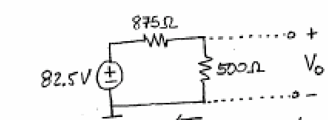
$P_{R_A} = 200 \mu W \Rightarrow V_A = \pm 1V \ldots (R_A = 5k\Omega)$

$\Rightarrow$ CHOOSE $\alpha = 10$ OR $\alpha = -22$

(BOTH WILL PROVIDE $200 \mu W$)

Question 9

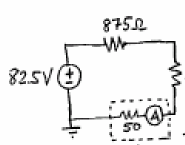
(a) REDUCE CIRCUIT: (e.g. SOURCE TRANSFORMATIONS)



$V_o = 500 I_o = 30V$

$I_o = 82.5/(875+500) = 60 mA$

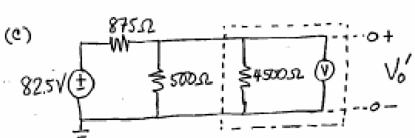
(b)



$I_o' = 82.5/(875+500+50) \equiv 57.9 mA$;

$100(I_o - I_o')/I_o \equiv 3.5\%$ ERROR

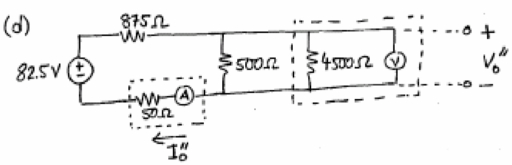
(c)



$V_o' = (500 \| 4500)(82.5)/(875 + 500\|4500) \equiv 28V$;

$100(V_o - V_o')/V_o \equiv 6.6\%$ ERROR

(d)



$I_o'' = 82.5/(875 + 500\|4500 + 50) = 60mA$;

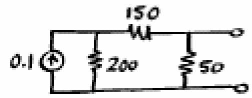
(ZERO ERROR!)

$$V_o'' = \frac{(500\|4500)(82.5)}{875 + 500\|4500 + 50} = 27V \;;$$ (10% ERROR

NOTE: I_o'' ERROR REDUCED FROM I_o' LEVEL (TO ZERO!) BECAUSE VOLTMETER LOADING EFFECT IS EXACTLY CANCELLED BY AMMETER LOADING EFFECT. V_o'' ERROR INCREASED FROM V_o' LEVEL BECAUSE AMMETER LOADING EFFECT ADDS TO THAT OF VOLTMETER (TO FURTHER DECREASE VOLTAGE DIVISION FOR 500Ω).

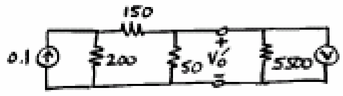
Question 10

1. REDUCE CIRCUIT USING SOURCE TRANSFORMATIONS :

⇒ 

$V_o = 50\left(\frac{200}{200+150+50}\right)0.1 = 2.5V$ //

FOR VOLTMETER :



$50 \rightarrow 50 \,||\, 5500 \approx 49.55\,\Omega$

$\Rightarrow V_o' \approx 2.48\,V$ //

Question 11

$v_+ = v_- = v_o/2$; $v_o = 40V$; $i_o = 140mA$ //

$P_{TOTAL} = P_{150} + P_{300} + 2P_{200} + P_{1000} + P_{10V} \approx 8.27W$ //

SOLVE $P_{150} + P_{300} + 2P_{200} + P_R = 10\,P_{10V}$ FOR R ;

$\Rightarrow R = \frac{40^2}{(2/3)} = 2400\,\Omega$ //