Department of Electrical and Computer Engineering

Computer Engineering

Course ECSE-322B

Problem Set 9 Solutions

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1. Most disk drives, both hard and soft, are designed to record over only part of the available surface.

(a) Explain why this is so.

Solution:

To maximize the amount of data stored on a disk. The total amount is controlled by the amount stored on the track with the minimum radius since the number of bytes per angular section of disk (sector) is the same for all tracks.

(b) Derive an expression for the inner radius, r, of the recording area of a disk (with an outer radius of R inches, t tracks per inch and a recording density of d bits per inch) such that the amount of data stored on the disk is maximized.

Solution:

Let the total amount of data on the disk be D.

*Then D = (R-r) * t * 2 * pi * r * d*

*= 2 * pi * d * t * r *(R-r)*

*Thus dD/dr = 2 * pi * d * t * (R - 2r)*

For a maximum amount of storage, this should be set equal to zero

Thus R - 2r = 0 or r = R/2

The amount of data then stored on the disk is:

*D = R ^2 * pi * t * d / 2*

(c) What would be the effect in terms of disk performance and amount of data stored if the inner radius derived in part (b) was halved?

Solution:

If the inner radius from part (b) is halved, there would be less data per track and more tracks. Thus the access time would go up as there would be more track to track head moves.

if $r = 0.25 R$

$$D = 0.75 * R * t * 2 * pi * 0.25 * R * d$$
$$= 0.375 R^2 * pi * t * d$$

Thus the total data stored is reduced by 25%

(d) What would be the effect in terms of disk performance and amount of data stored if the inner radius derived in part (b) was increased by 50%?

Solution:

Increasing the inner radius will increase the data stored per track and result in fewer tracks. Thus the total data stored on the disk will be reduced, but the number of track to track head moves will also reduce leading to faster access times.

if $r = 0.75 R$

$$D = 0.25 * R * t * 2 * pi * 0.75 * R * d$$
$$= 0.375 R^2 * pi * t * d$$

Thus there is a reduction over the maximum possible data storage of 25%

2. Why is it advisable to defragment a disk and what causes fragmentation in the first place?

Solution:

Defragmenting a disk will restructure the files on a disk so that the access times are minimized. It is advisable to do this to keep the disk operating at its maximum possible performance.

Fragmentation occurs because of the linked list structure used to store files. As new files are written to the disk and old files are deleted, files end up in non-sequential storage and any given file may occupy sectors on several tracks of the disk. This can result in excessive head moves to access a file and a resulting slow down in file access.

3. A common disk size is 3.5 inches in diameter. If the innermost track is recorded 0.75 inches from the centre and the outermost track is recorded 1.65 inches from the centre, what is the density of bits on the surface of the disk? Assume the surface contains 150 megabytes of data. What assumption did you make?

Solution:

From the solutions to question 1, the amount of data on the disk is:

$$d * 2 * pi * 0.75 * (1.65-0.75) * t$$

hence

$$150 \text{ Mbytes} = 2 * pi * 0.675 * d * t$$

Now, the linear density of bits around a track is d and the number of tracks per inch is t. If it assumed that the density being recorded is the product of the two (i.e. it is not the linear density d squared) then the density is given by

$$density = 150 * 10^6 * 8/ (2 * pi * 0.675) \text{ bits per square inch (8 bits per byte)}$$

$$density = 282.94 \text{ Mbits per square inch}$$

4. What are the access times of semiconductor memory, hard disks, and floppy disks? (i.e. determine the “order” of the timing)

Solution:

The access time of semiconductor memory is of the order of nanoseconds

The access time of a magnetic hard disk is of the order of milliseconds

The access time of a floppy disk is of the order of seconds

5. Why do disks and tapes require that the media move past the read/write heads in order for data to be read?

Solution:

> *The read operation of any magnetic storage system is based on the principle embodied in Faraday's Law. This requires that the reading system (the read head for a tape or a disk) sees a change in flux with time. This is obtained in a tape or disk system by moving the magnetized sections of the disk or tape past the head.*

6. A disk has 250 tracks and rotates at 2400 rpm. The average time for moving the head between adjacent tracks is 0.4 ms. What are the average seek and latency times of that disk, approximately?

Solution:

2400 rotations per minute = 40 rotations per second, thus a rotation takes 25 milliseconds

On average, the rotational delay is 0.5 a rotation. Thus the average rotational latency is:
12.5 milliseconds

If it assumed that all seeks start from track 0, the average seek time will be:
*(249/2) * 0.4 = 49.8 milliseconds*

However, if the seek time is computed based on starting from a random track, the following is true:

Assume that the head can start on a track, i, where i = 1,2,3,4,...
The average seek time from i to any other track of the disk is:

to track i - 1: the time is t
to track i - 2: the time is 2t
...
*to track 1: the time is (i-1)*t*

Going to the higher numbered tracks, the time is:

to track i + 1: the time is t
to track i + 2: the time is 2t
...
*to track N: the time is (N-i)*t*

Thus the sum of all these times is:

$$T = t\left(\sum_{n=1}^{i-1} n + \sum_{n=1}^{N-1} n\right)$$

Expanding and simplifying the above expression gives:

$$T = t(2i^2 + N^2 - 2Ni - 2i + N)/2$$

There are N-1 possible number of tracks to move to so the average time from track i is:

$$T = t(2i^2 + N^2 - 2Ni - 2i + N)/[2(N-1)]$$

Now we need to consider the above sum for each value of i:

i.e. i = 1,2,3,4,...,N and divide by the total number of tracks, N, to obtain a full expression for the average access time.

$$Average\ time = [t / 2N(N-1)]\sum_{i=1}^{N}(2i^2 + N^2 - 2Ni - 2i + N)$$

Since the sum of the i^2 terms is N(N+1)(2N+1)/6, and the sum of the i terms is N(N+1)/2, the overall sum is:

$$N(N+1)(2N+1)/3 + N^2(N) - N^2(N+1) - N(N+1) + N(N)$$

which reduces to:

$$[2N(N+1)(N-1)]/3$$

Substituting into the average time expression and simplifying yields:

$$Average\ time = [(N+1)t]/3$$

For large N, this expression is approximately Nt/3.

*Thus, for a random track, the average seek time is 250*0.4/3 = 33.33 milliseconds*

7. A disk rotates at 3000 rpm and has an average seek time of 70 ms.What is the average time needed to read a file that is stored in 12 sectors, assuming that each of these sectors is on a different track?

Solution:

The average time is the sum of the average seek time and the average rotational latency.
The average seek time is 70ms.
3000 rpm = 50 rev/sec or 20 ms/rev.
On average, the time to find and read the appropriate sector in a track will be about half a revolution or 10 ms.

*Thus the time to read the file will be 11*70 + 12*10 assuming the head is positioned over the first track of the file at the time that the read begins (but is not over the correct sector).*

This gives a total time of 0.89 seconds

8. A computer system is designed to accept several 3.5 inch hard disk drives. The disks are state of the art and use a recording density of 800 Mbits/square inch. One surface on each disk is to be used for timing information and thus is unavailable for storing data and the disks are formatted to store 1024 bytes per sector. The disks are designed to use a recording area which has an inner radius of 0.75 inches and an outer radius of 1.75 inches. If the address structure for accessing the disk drives is as follows:

3 bits for the disk address
4 bits for the surface address
11 bits for the track address
4 bits for the sector address,

what is the maximum amount of disk space that can be installed on the computer?

Solution:

From the track address, these disks have 2^{11} or 2048 tracks and these are stored over an annulus 1 inch in width (1.75 - 0.75).

Each surface has 1024 bytes in a sector and, from the addressing scheme, there are 16 sectors per track.

*Thus the total data per surface is 1024 * 16 * 2048 = 32 Mbytes.*

There are 16 possible surfaces per drive but one is used for timing so a complete drive has

*32 * 15 = 480 Mbytes*

Finally, from the last part of the address, the computer can address up to 8 drives, so the maximum amount of disk is:

*8 * 480 = 3840 Mbytes*

Note that the recording density is not actually needed in this question - the actual recording density on the disk is 774 Mbits per square inch (from the linear density squared).