COMP 250 : Exercise Set #2, Part #2 : Feb. 16, 2009 : Recurrences and Induction
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1 Recurrences

1. Solve the recurrence, for positive constant c,

$$T(n) = T(\frac{n}{2}) + c \cdot \log n$$

2. Solve the recurrence , where $T(1) = 1$,

$$T(n) = 2T(n-1) + 2$$

3. Solve the recurrence, with $T(1) = 1$

$$T(n) = 5T(\frac{n}{5}) + 2n$$

2 Induction

1. By induction (that means, don't use a formula you already know in the proof), show that

$$1 + 3 + 5 + 7 + \cdots + (2n-1) = n^2$$

2. In class, you encountered the Fibonacci Sequence, defined as

$$F_n = F_{n-1} + F_{n-1}$$

with initial values $F_1 = 1$ and $F_2 = 1$.

You saw several ways to calculuate this, including recursively, and with matrices.

Another way is by calculating the following exact expression for the Fibonacci number

$$F_n = \frac{1}{\sqrt{5}} \left[\left(\frac{1+\sqrt{5}}{2} \right)^n - \left(\frac{1-\sqrt{5}}{2} \right)^n \right]$$

Prove it by induction.

HINT: You will need to prove two base cases, F_1 and F_2, as the definition of F_n goes "back" two in the sequence.

HINT: It may be useful to let $\phi = \frac{1}{2}(1+\sqrt{5})$ and $\tau = \frac{1}{2}(1-\sqrt{5})$, and observe that both ϕ and τ satisfy the equation $x^2 = x + 1$.
This would re-write F_n as

$$\frac{1}{\sqrt{5}} \left(\phi^n - \tau^n \right)$$

, and make your algebra much simpler. You can then use the relation $x^2 = x + 1$ to simplify the expression.

ENCOURAGEMENT: I really recommend trying this WITHOUT looking at the solution(which I provided in full). It will likely take some messing around, and may be annoying. But it will be good for you!