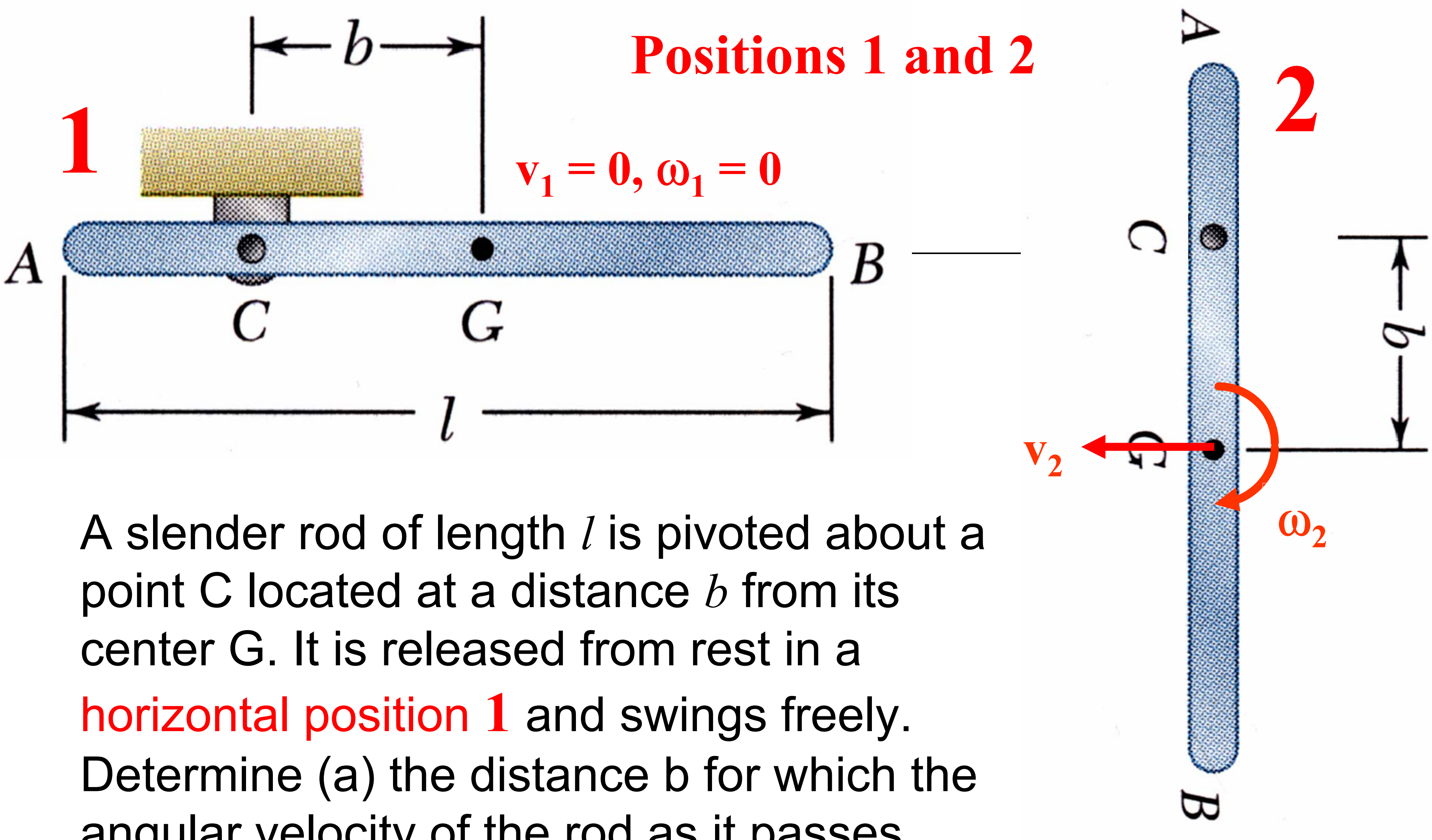
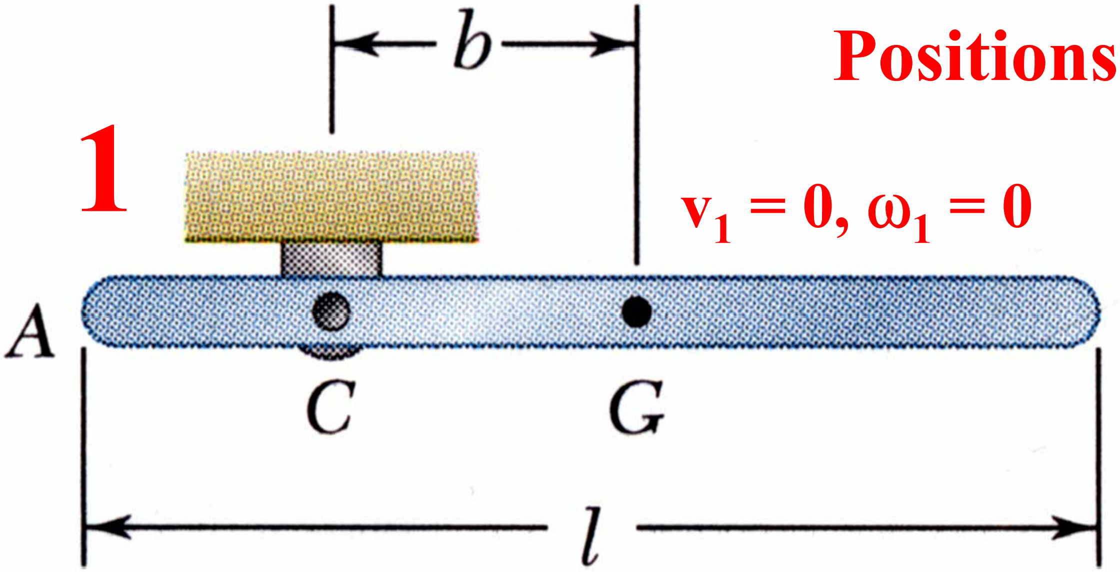
Positions 1 and 2

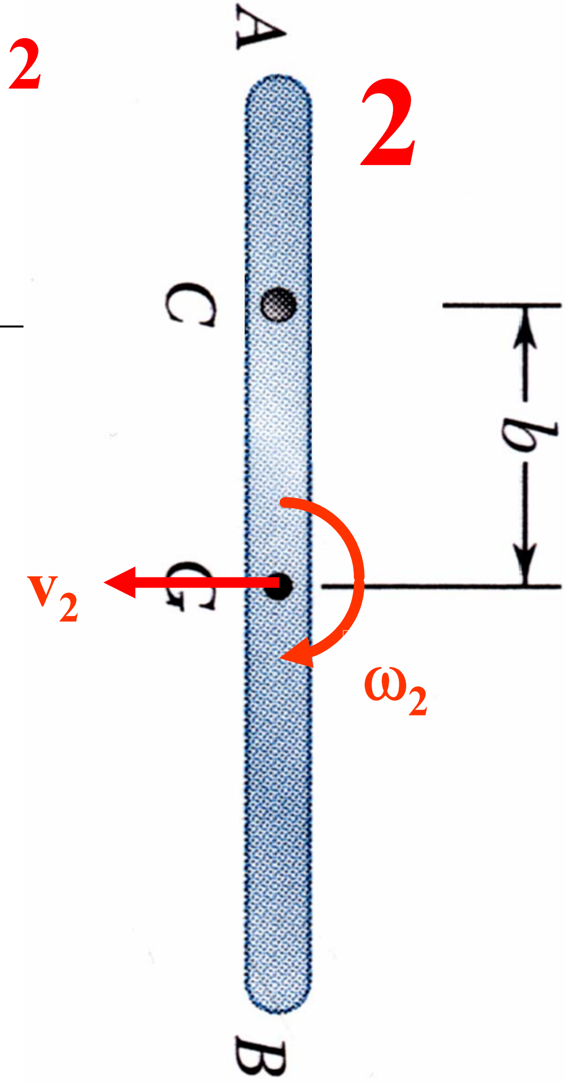


A slender rod of length *l* is pivoted about a point C located at a distance *b* from its center G. It is released from rest in a horizontal position **1** and swings freely. Determine (a) the distance b for which the angular velocity of the rod as it passes through a vertical position **2** is maximum, (b) the corresponding values of its angular velocity and of the reaction at C.

Positions 1 and 2



$v_1 = 0, \omega_1 = 0$



v_2, ω_2

$$T_1 = 0$$

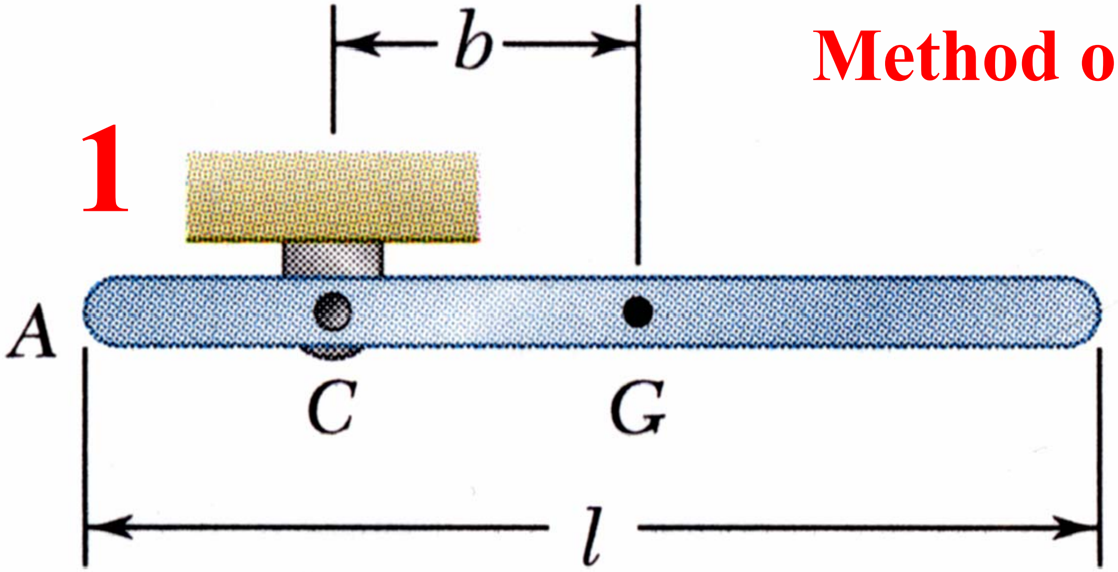
$$T_2 = \frac{1}{2} m \bar{v}_2^2 + \frac{1}{2} \bar{I} \omega_2^2$$

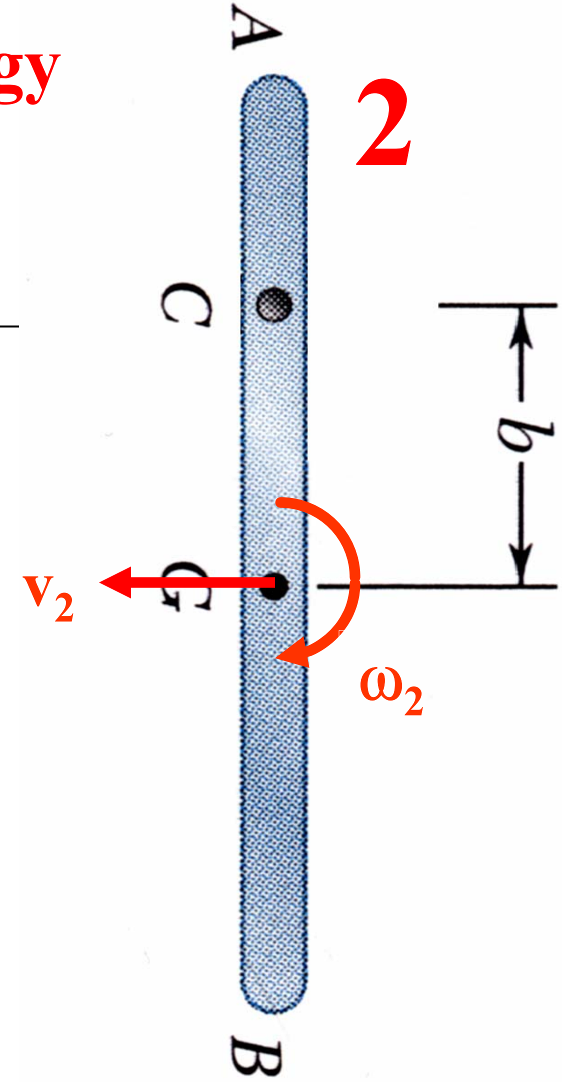
$$= \frac{1}{2} m (b\omega_2)^2 + \frac{1}{2} \left(\frac{1}{12} m \ell^2\right) \omega_2^2 = \frac{1}{2} m \left(b^2 + \frac{\ell^2}{12}\right) \omega_2^2$$

Kinematics:

$v_2 = b\, \omega_2$

Method of Energy





$$T_1 = 0 \qquad T_2 = \frac{1}{2}m\left(b^2 + \frac{\ell^2}{12}\right)\omega_2^2$$

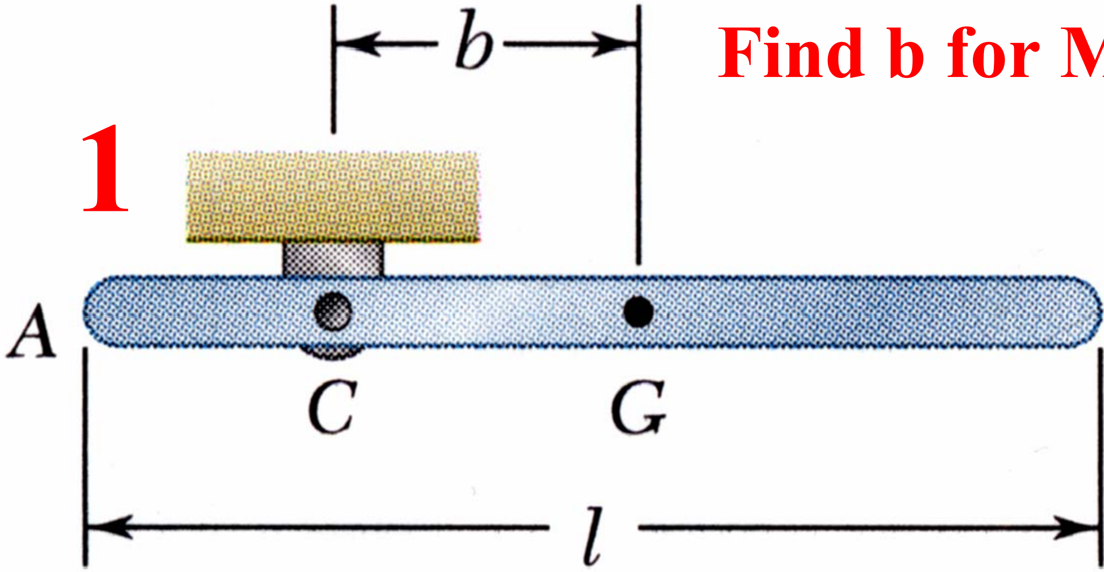
$$V_1 = 0 \qquad V_2 = -mgb$$

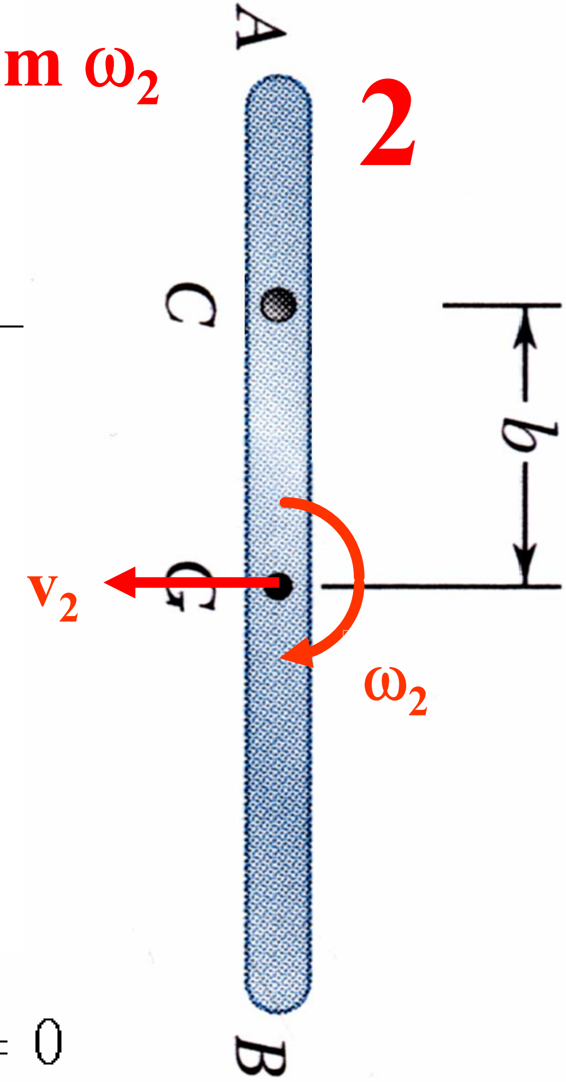
$$\omega_2^2 = \frac{2gb}{b^2 + \frac{1}{12}\ell^2}$$

Kinematics:

$v_2 = b\ \omega_2$

Find b for Maximum ω_2





$$\omega_2^2 = \frac{2gb}{b^2 + \frac{1}{12}\ell^2}$$

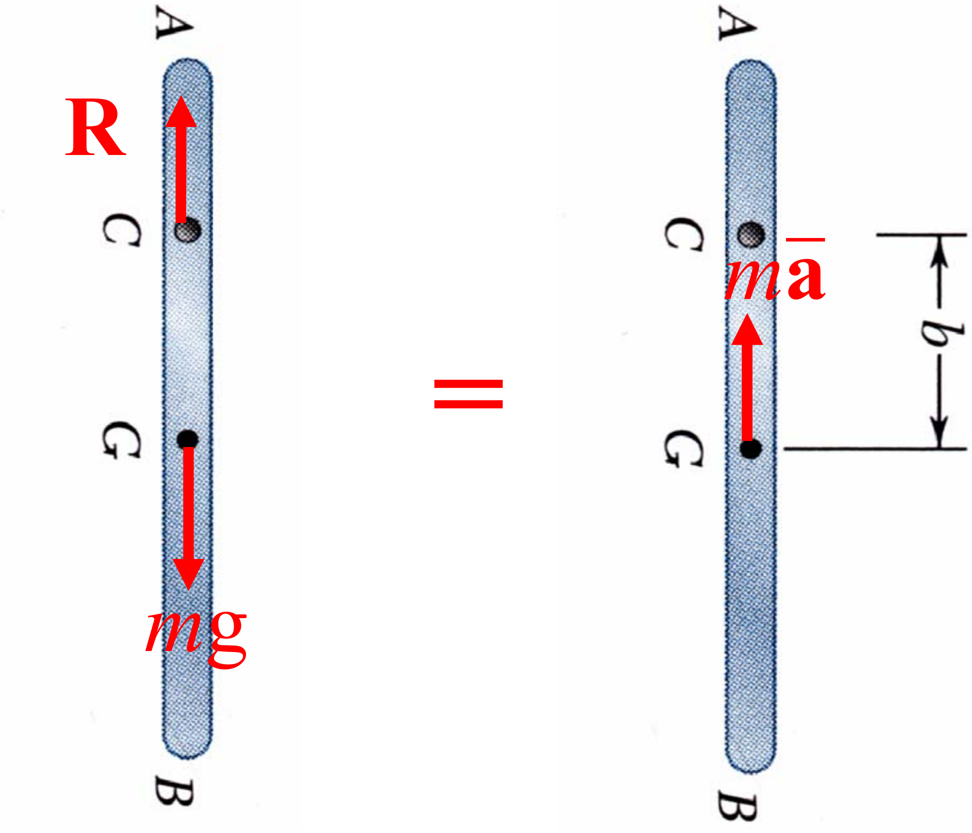
Maximum ω_2 :

$$\frac{d}{db}(\omega_2^2) = \frac{2g}{[b^2 + \frac{1}{12}\ell^2]^2}[(b^2 + \frac{1}{12}\ell^2) - b(2b)] = 0$$

$$-b^2 + \frac{1}{12}\ell^2 = 0.$$

Hence, $b = \frac{\ell}{\sqrt{12}}$ $\quad \omega_2^2 = \sqrt{12}\frac{g}{\ell}$

Reaction Force at C



$$b = \frac{\ell}{\sqrt{12}} \qquad \omega_2^2 = \sqrt{12}\frac{g}{\ell}$$

$$\uparrow \quad R - mg = m\,\bar{a}$$

$$\bar{a} = b\omega_2^2 = g \qquad \text{(centripedal)}$$

$$\boxed{R = 2\,m\,g}$$