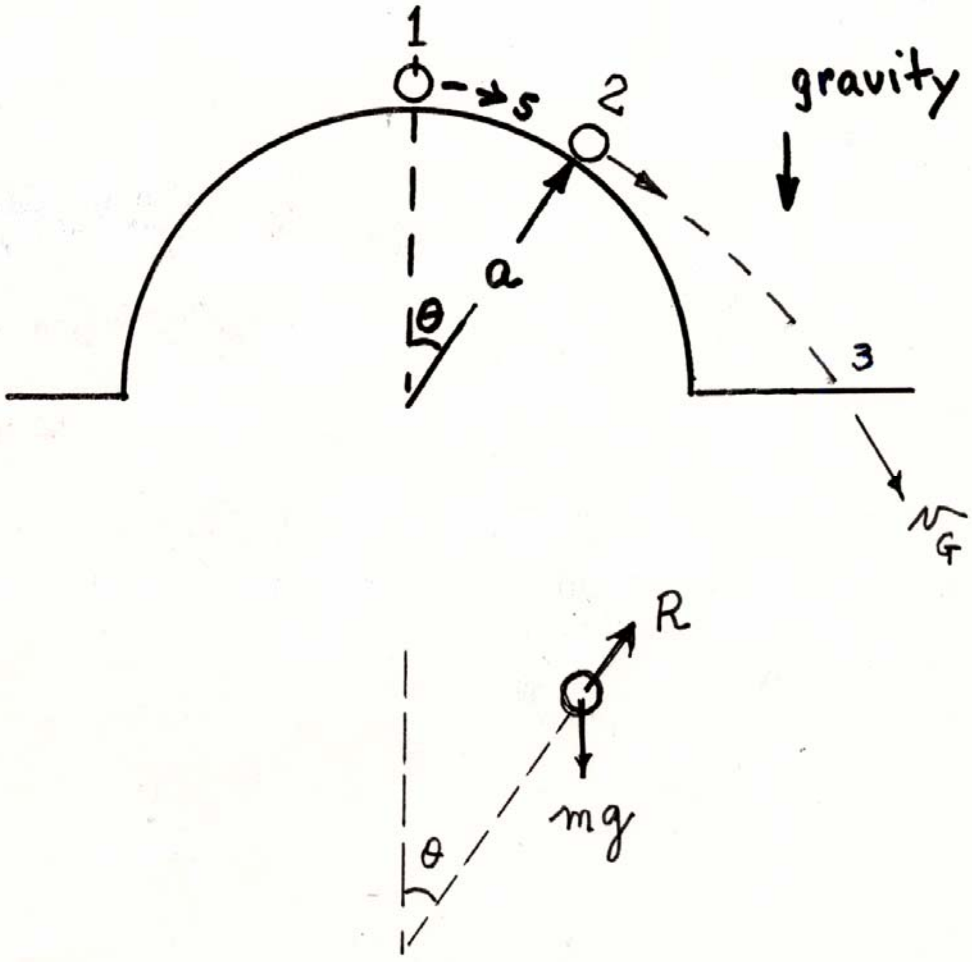
A small mass, initially at rest on the top of a smooth hemi-spherical surface, slides down under gravity. (a) Find the position where the mass begins to leave the surface. (b) Find the velocity when the mass hits the floor.

1-2: slides along the surface

Find R(θ) and set R to zero to determine the critical angle θ_c at 2.

2-3: projectile

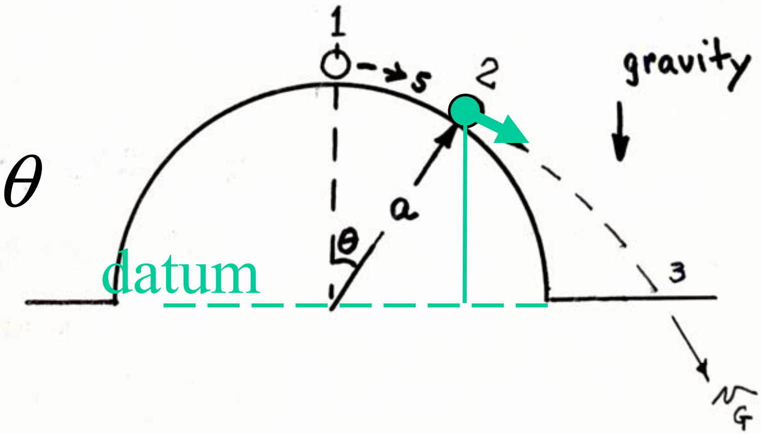
Use energy equation to find the velocity at 3.



Energy Equation: $T_1+V_1=T_2+V_2$

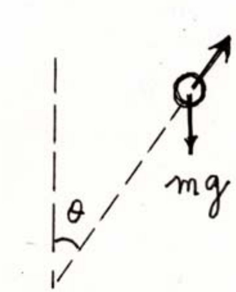
$$\tfrac{1}{2} m\, v_1^2 + mg\,a = \tfrac{1}{2} m\, v_2^2 + mg\,a\cos\theta$$

$$v_2^2 = 2ga\,(1-\cos\theta)$$



$F = ma$ in the radial direction:

$$mg\cos\theta - R = m\,a_n = m\,v^2/a$$



At 2, $R = 0$:

$$mg\cos\theta_2 = m\,v^2/a = 2mg\,(1-\cos\theta_2)$$

$$\cos\theta_2 = 2\,(1-\cos\theta_2) \qquad \cos\theta_2 = \tfrac{2}{3}$$

$$\boxed{v_2^2 = \tfrac{2}{3}\,ga}$$

2-3: projectile problem

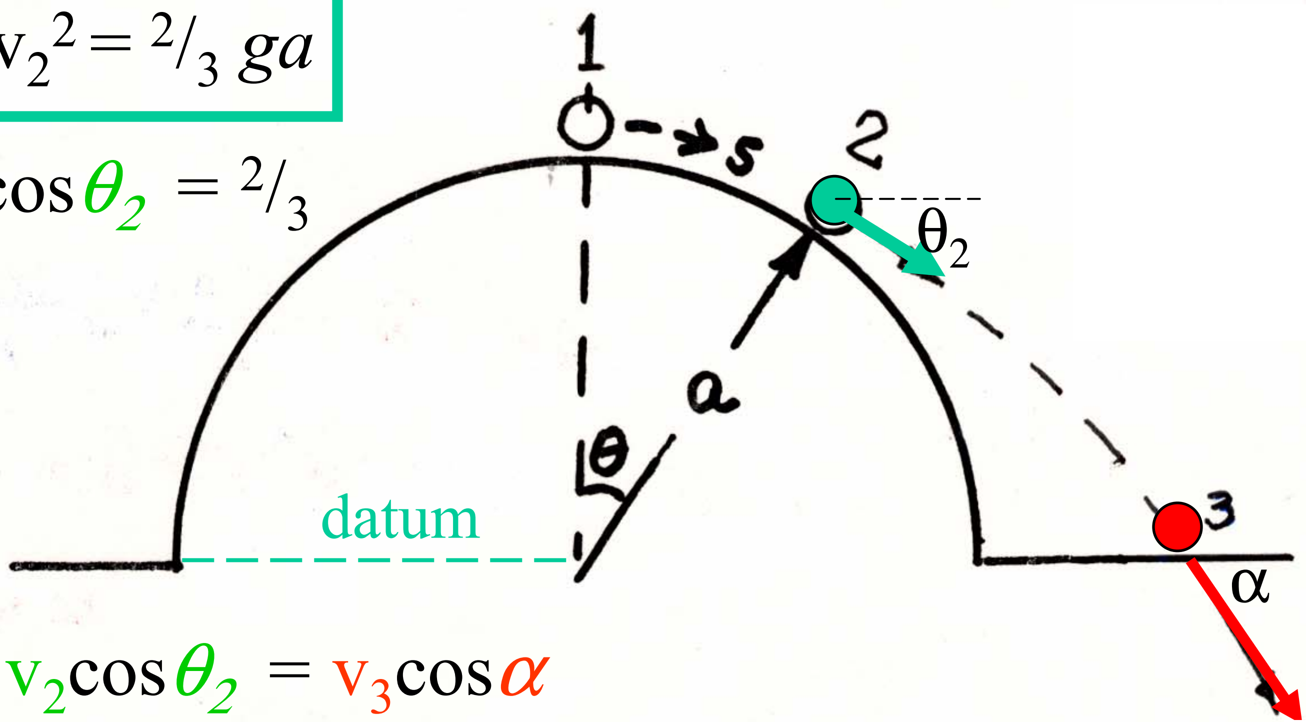
Energy Equation: $T_1+V_1=T_3+V_3$

$$v_3^2 = 2\,ga$$

$$0 + mg\,a = {}^{1}\!/_{2}\, m\, v_3^2 + 0$$

$$v_2^2 = {}^{2}\!/_{3}\, ga$$

$$\cos\theta_2 = {}^{2}\!/_{3}$$



$$v_2\cos\theta_2 = v_3\cos\alpha$$

$$v_3\sin\alpha - v_2\sin\theta_2 = g\,t_{2\text{-}3}$$

$$x_{2\text{-}3} = t_{2\text{-}3}\, v_2\cos\theta_2$$