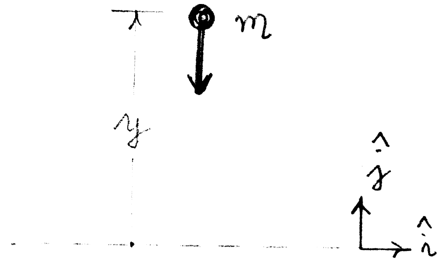
Examples of Conservative Forces and its Potentials

Uniform Gravitational Force



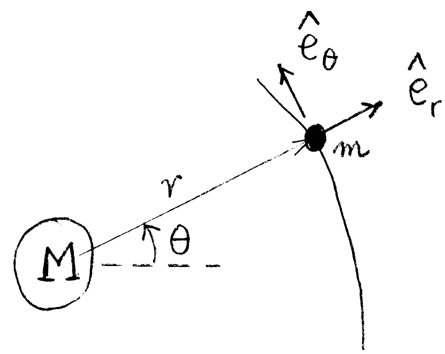
$$\mathbf{F} = -mg\hat{\mathbf{j}}$$

$$d\mathbf{r} = dx\hat{\mathbf{i}} + dy\hat{\mathbf{j}} + dz\hat{\mathbf{k}}$$

Gravity potential is the negative work by $\mathbf{F}$:

$$V(r) = -\int^{\mathbf{r}} \mathbf{F} \cdot d\mathbf{r} = -\int^{y} (-mg)dy = mgy$$

Universal Law of Gravitation



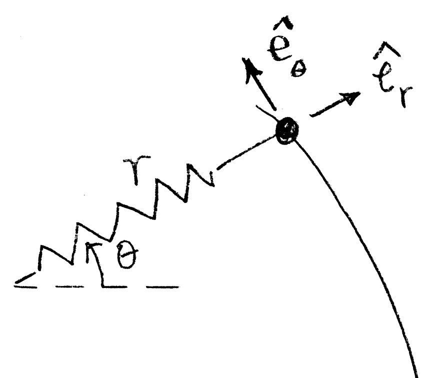
$$\mathbf{F} = -\frac{GMm}{r^2}\hat{\mathbf{e}}_r$$

Note that $\mathbf{r} = r\hat{\mathbf{e}}_r$ and that

$$d\mathbf{r} = dr\hat{\mathbf{e}}_r + r\frac{d\hat{\mathbf{e}}_r}{d\theta}d\theta = dr\hat{\mathbf{e}}_r + rd\theta\hat{\mathbf{e}}_\theta.$$

The gravity potential is

$$V(r) = -\int^{\mathbf{r}} \mathbf{F} \cdot d\mathbf{r} = -\int^{r} (-\frac{GMm}{r^2})dr = -\frac{GMm}{r}$$



Linear Spring

$$\mathbf{F} = -k(r-\ell)\hat{\mathbf{e}}_r$$

The force potential of the linear spring is

$$V(r) = -\int^{\mathbf{r}} \mathbf{F} \cdot d\mathbf{r} = -\int^{r} [-k(r-\ell)]dr = \frac{1}{2}k(r-\ell)^2 = \frac{1}{2}k\xi^2$$

where k = stiffness of the spring and $\xi = r - \ell$ = the length of the extension or compression.