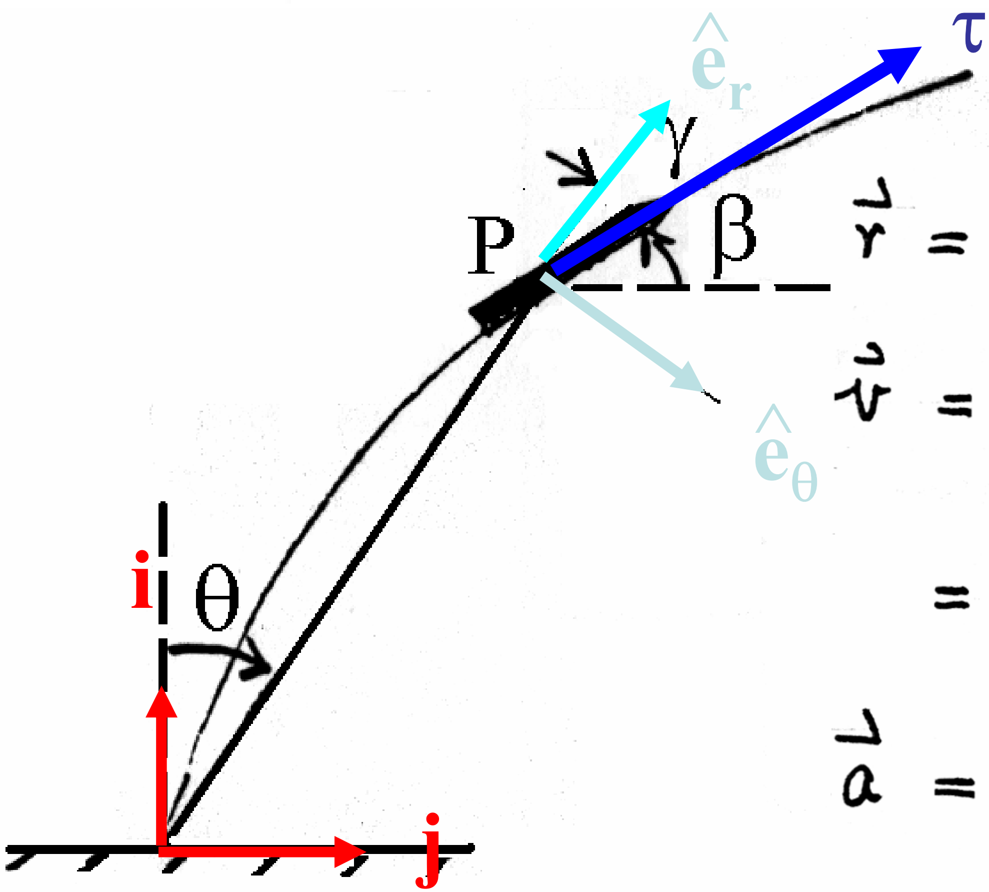
Example 3: A rocket is tracked by radar from its launching point A. When it is 10 second into its flight, the following radar measurements are recorded: $r = 2200$ m, $\dot{r} = 500$ m/s, $\ddot{r} = 4.66$ m/s^2, $\theta = 22°$, $\dot{\theta} = 0.0788$ rad/s, $\ddot{\theta} = -0.0341$ rad/s^2. (a) Determine the angle β between the horizontal and the direction of the trajectory of the rocket. (b) Find the magnitude of its velocity, v, and the acceleration, a. (c) Find the radius of curvature of the trajectory at this position. Note that $\hat{e}_\theta$ is defined as the based vector in the direction of increasing θ which, in this case, is measured from the vertical.



$$\vec{r} = r\,\hat{e}_r$$

$$\vec{v} = \dot{r}\,\hat{e}_r + r\,\frac{d\hat{e}_r}{d\theta}\cdot\dot{\theta}$$

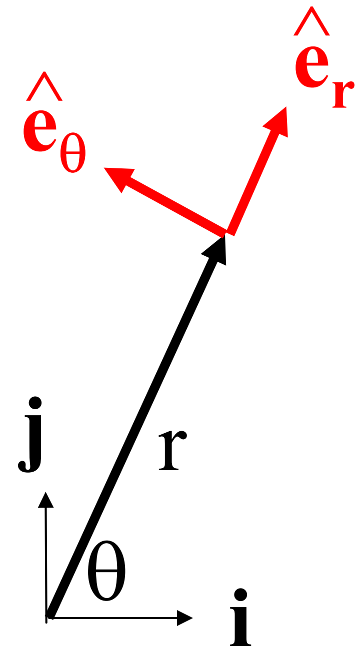
$$= \dot{r}\,\hat{e}_r + r\,\dot{\theta}\,\hat{e}_\theta$$

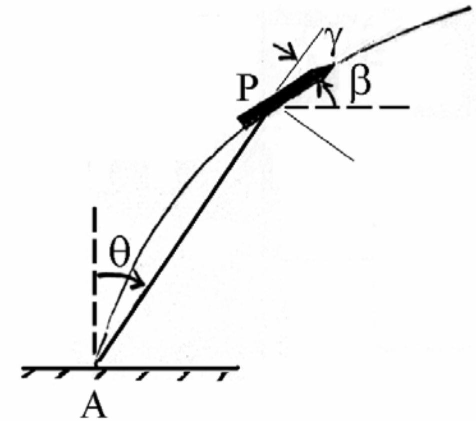
$$\vec{a} = (\ddot{r} - r\,\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta$$

Cylindrical vs Cartesian Coordinates

$$\mathbf{i} = \text{Cos}\,\theta\;\hat{\mathbf{e}}_r - \text{Sin}\,\theta\;\hat{\mathbf{e}}_\theta$$

$$\mathbf{j} = \text{Sin}\,\theta\;\hat{\mathbf{e}}_r + \text{Cos}\,\theta\;\hat{\mathbf{e}}_\theta$$



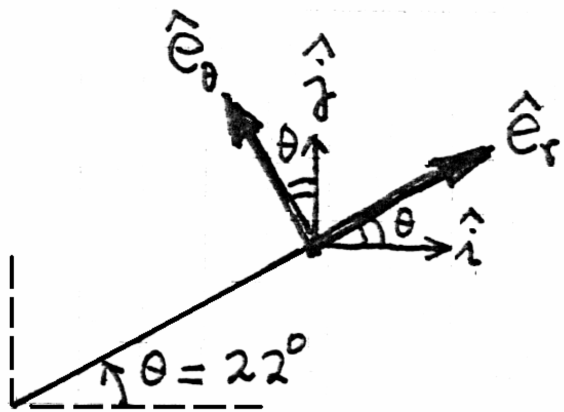


$$\vec{v} = \dot{r}\,\hat{e}_r + r\dot{\theta}\,\hat{e}_\theta = 500\,\hat{e}_r + 173.4\,\hat{e}_\theta$$

$$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{(500)^2 + (173.4)^2} = 529.2 \ \text{m/s}$$

$$\hat{\tau} = \frac{\vec{v}}{v} = 0.9448\,\hat{e}_r + 0.3277\,\hat{e}_\theta$$

$$\hat{\tau}\cdot\hat{j} = \cos\beta = (0.9448\,\hat{e}_r + 0.3277\,\hat{e}_\theta)(\sin\theta\,\hat{e}_r + \cos\theta\,\hat{e}_\theta)$$



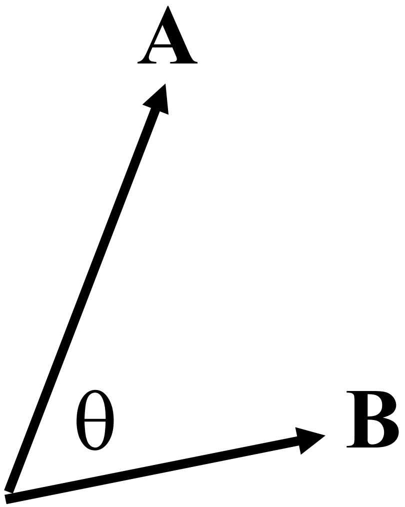
$$= (0.9448,\ 0.3277)\cdot(0.3746,\ 0.9272)$$

$$= 0.9448\times 0.3746 + 0.3277\times 0.9272$$

$$= 0.65776 \longrightarrow \beta = 48.87^\circ$$

Scalar Product or Dot Product

$$\mathbf{A}\cdot\mathbf{B} = A\ B\ \mathrm{Cos}\ \theta$$



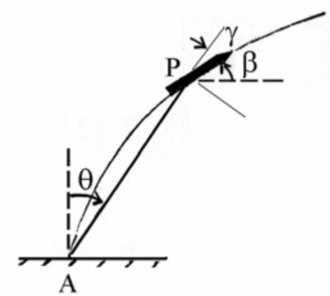
$$a_r = \ddot{r} - r\dot{\theta}^2 = 4.66 - 2200(0.0788)^2 = -9.00 \ m/s^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2200(-0.0341) + 2\times 500 \times 0.0788 = 3.78 \ m/s^2$$

$$a_t = \vec{a}\cdot\hat{t} = (-9.00, 3.78)\cdot(0.9448, 0.3277) = -7.264 \ m/s^2$$

$$a^2 = a_r^2 + a_\theta^2 = (9.00)^2 + (3.78)^2$$

$$v^2 = v_r^2 + v_\theta^2 = (500)^2 + (173.4)^2$$



$$a_n = \sqrt{a^2 - a_t^2} = \sqrt{(9.00)^2 + (3.78)^2 - (7.264)^2} = 6.521 \ m/s^2$$

$$\rho = \frac{v^2}{a_n} = \frac{(500)^2 + (173.4)^2}{6.521} = 4.29\times 10^4 \ m = 42.9 \ km$$