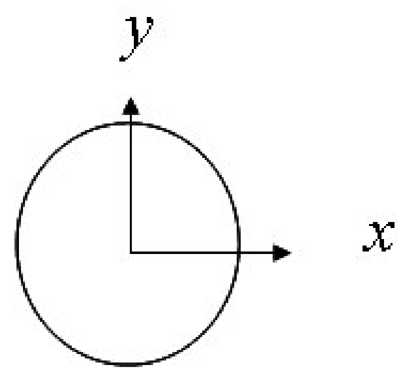
EXAMPLE: The position of a particle is given in the Cartesian coordinates as a function of time as follows:

$$x = \sin 2\pi t, y = \cos 2\pi t, z = 0$$

Decribes the trajectory of the particle.
Find the velocity and acceleration vectors of the particle.



- The particle trajectory is a circle because $x^2 + y^2 = 1$.
- The position vector has three components. In the Cartesian coordinates, the vector is

$$\mathbf{r} = (x, y, z) = (\sin 2\pi t, \cos 2\pi t, 0)$$

- The velocity vector is the component-by-component time differentiation of the position vector:

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = (v_x, v_y, v_z) = (2\pi \cos 2\pi t, -2\pi \sin 2\pi t, 0) = (v_x, v_y, v_z)$$

The dot (scalar) product of **v** and **r** is

$$\mathbf{v} \cdot \mathbf{r} = v_x x + v_y y + v_z z = 2\pi \cos 2\pi t(\sin 2\pi t) - 2\pi \sin 2\pi t(\cos 2\pi t) = 0$$

In this example, the velocity vector **v** is in a direction perpendicular to the position vector **r**; **r** is in the radial direction and **v** is in a direction tangential to the path of the motion.

- The accelation vector is the time differentiation of the velocity vector:

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = -(2\pi)^2(\sin 2\pi t, \cos 2\pi t, 0)$$

which is a vector in the oposite direction of the position vector. In this case, the acceleration vector has only a normal component; the tengential component of the acceleration is zero.