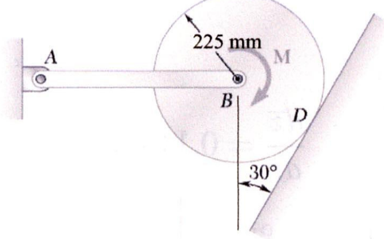
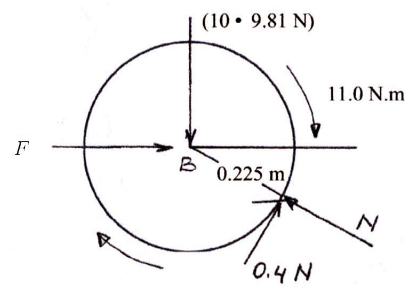
PROBLEM 16.23



A 10 kg uniform disk is placed in contact with an inclined surface and a constant 11.0 N.m couple **M** is applied as shown. The weight of the link *AB* is negligible. Knowing that the coefficient of kinetic friction at *D* is 0.4, determine (*a*) the angular acceleration of the disk, (*b*) the force in the link *AB*.

SOLUTION



$$I\alpha = \frac{1}{2}(10\,\text{kg})(0.225\,\text{m})^2\alpha = (0.2531\,\text{kg.m}^2)\alpha$$

$$+\circlearrowright\ \Sigma M_B = 11.0\text{ N.m} - (0.4N)(0.225\,\text{m}) = \frac{1}{2}(10\text{ kg})(0.225\text{ m})^2\alpha$$

$$+\uparrow \Sigma F_y = 0.4N\cos 30° + N\cos 60° - (10 \cdot 9.81\text{ N}) = 0;\quad N = 115.903\text{ N}$$

$$\alpha = \frac{11.0\text{N.m} - (0.4)(115.903\text{N})(0.225\text{m})}{I}$$

$$= \frac{0.56873\text{N.m}}{0.2531\text{kg.m}^2} = 2.247\text{ rad/s}^2$$

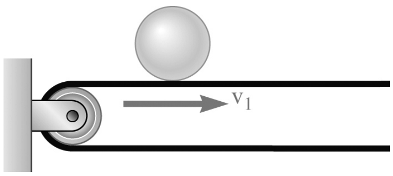
(*a*) $\alpha = 2.25\text{ rad/s}^2$ ⟳ ◀

$$\overset{+}{\longrightarrow} \Sigma F_x = F - (0.866)N + 0.4N(0.5) = 0$$

$$F = 0.6660N = (0.6660)(115.903\text{ N})$$

$$= 77.19\text{ N}$$

(*b*) or $F = 77.2$ N (compression) ◀

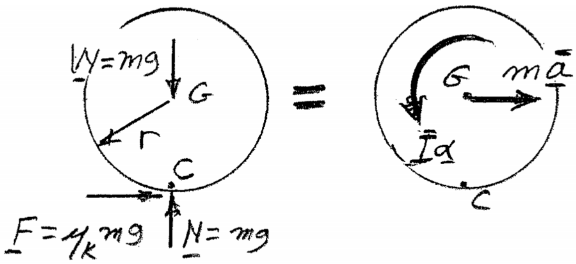


PROBLEM 16.68

A uniform sphere of radius r and mass m is placed with no initial velocity on a belt that moves to the right with a constant velocity $\mathbf{v}_1$. Denoting by μ_k the coefficient of kinetic friction between the sphere and the belt, determine (*a*) the time t_1 at which the sphere will start rolling without sliding, (*b*) the linear and angular velocities of the sphere at time t_1.

SOLUTION

Kinetics:



$$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad F = m\bar{a}$$

$$\mu_k mg = m\bar{a}$$

$$\bar{\mathbf{a}} = \mu_k g \longrightarrow$$

$$+\circlearrowleft \Sigma M_G = \Sigma(M_G)_{\text{eff}}: \quad Fr = \bar{I}\alpha$$

$$(\mu_k mg)r = \frac{2}{5}mr^2\alpha$$

$$\boldsymbol{\alpha} = \frac{5}{2}\frac{\mu_k g}{r} \circlearrowleft$$

Kinematics:

$$\xrightarrow{+} \bar{v} = \bar{a}t = \mu_k gt \quad (1)$$

$$+\circlearrowleft \omega = \alpha t = \frac{5}{2}\frac{\mu_k g}{r}t \quad (2)$$

C = Point of contact with belt

$$\xrightarrow{+} v_C = \bar{v} + \omega r = \mu_k gt + \left(\frac{5}{2}\frac{\mu_k g}{r}t\right)r$$

$$v_C = \frac{7}{2}\mu_k gt$$

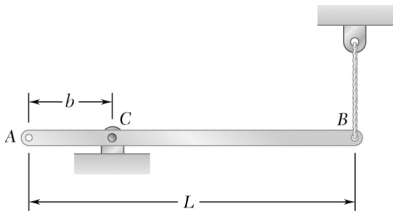
(*a*) When sphere starts rolling $(t = t_1)$, we have

$$v_C = v_1; \qquad v_1 = \frac{7}{2}\mu_k gt_1 \qquad\qquad t_1 = \frac{2}{7}\frac{v_1}{\mu_k g} \blacktriangleleft$$

(*b*) Velocities when $t = t_1$

Eq (1): $$\bar{v} = \mu g\left(\frac{2}{7}\frac{v_1}{\mu_k g}\right) \qquad\qquad \bar{\mathbf{v}} = \frac{2}{7}v_1 \longrightarrow \blacktriangleleft$$

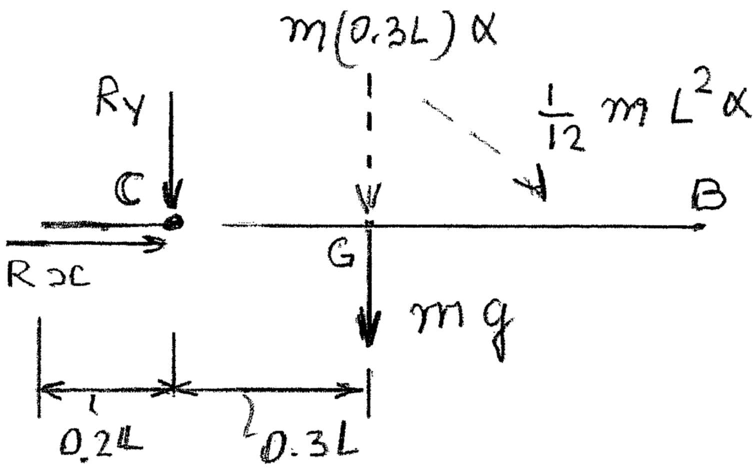
Eq (2): $$\omega = \left(\frac{5}{2}\frac{\mu_k g}{r}\right)\left(\frac{2}{7}\frac{v_1}{\mu_k g}\right) \qquad\qquad \boldsymbol{\omega} = \frac{5}{7}\frac{v_1}{r} \circlearrowleft \blacktriangleleft$$



PROBLEM 16.80

A uniform rod of length L and mass m is supported as shown with $b = 0.2\,L$ when the cable attached to end B suddenly breaks. Determine at this instant (*a*) the acceleration of end B, (*b*) the reaction at the pin support.

SOLUTION



$$+\circlearrowright\ \Sigma M_C = mg(0.3L) = \frac{mL^2\alpha}{12} + m(0.3L)^2\alpha$$

$$\alpha = \frac{0.3g}{0.1733L} = 1.73077\frac{g}{L} = \frac{45}{26}\frac{g}{L}$$

(*a*)

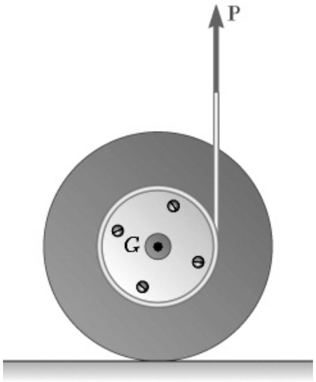
$$a_B = 0.8L\alpha = \frac{36g}{26}$$

$$\text{or}\quad a_B = \frac{18g}{13}\downarrow \blacktriangleleft$$

(*b*) $R_x = 0$

$$+\uparrow\ \Sigma F_y = R_y - mg = -0.3mL\left(\frac{45g}{26L}\right)$$

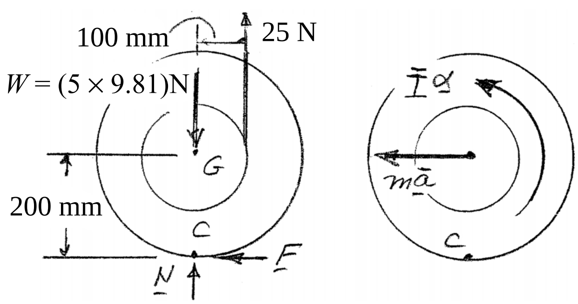
$$R_y = mg - \frac{135}{260}mg = \frac{25}{52}mg\uparrow \blacktriangleleft$$



PROBLEM 16.100

A drum of 100 mm radius is attached to a disk of 200 mm radius. The disk and drum have a total weight of 5 kg and combined radius of gyration of 150 mm. A cord is attached as shown and pulled with a force **P** of magnitude 25 N. Knowing that the disk rolls without sliding, determine (*a*) the angular acceleration of the disk and the acceleration of *G*, (*b*) the minimum value of the coefficient of static friction compatible with this motion.

SOLUTION



$$\bar{a} = r\alpha = \left(\frac{200}{1000}\text{m}\right)\alpha = (0.2\text{ m})\alpha$$

$$\bar{I} = m\bar{k}^2 = (5\text{ kg})(0.15\text{ m})^2$$

$$= 0.1125\text{ kg.m}^2$$

$$+\circlearrowleft \Sigma M_C = \Sigma (M_C)_{\text{eff}}: (25\text{ N})(0.1\text{ m}) = m\bar{a}r + \bar{I}\alpha$$

$$2.5\text{ N.m} = (0.3125\text{ kg.m}^2)\alpha$$

$$\alpha = 8\text{ rad/s}^2$$

or $\boldsymbol{\alpha} = 8\text{ rad/s}^2 \circlearrowleft$ ◀

(*a*) $\bar{a} = r\alpha = (0.2\text{ m})(8\text{ rad/s}^2) = 1.6\text{ m/s}^2$

or $\bar{\mathbf{a}} = 1.6\text{ m/s}^2 \leftarrow$ ◀

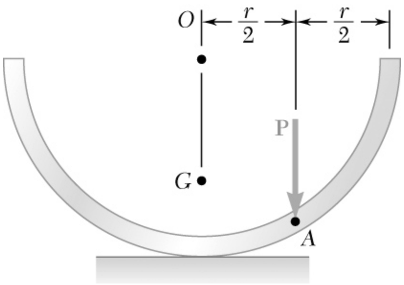
(*b*) $+\uparrow F_y = 0: \quad N + 25\text{ N} - (5 \times 9.81)\,N = 0 \quad N = 24.05\text{ N}$

$$\overset{+}{\leftarrow} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \quad F = m\bar{a} = (5\text{ kg})(1.6\text{ m/s}^2)$$

$$F = 8\text{ N}$$

$$(\mu_s)_{\min} = \frac{F}{N} = \frac{8\text{N}}{24.05\text{N}} = 0.3326\text{ N}$$

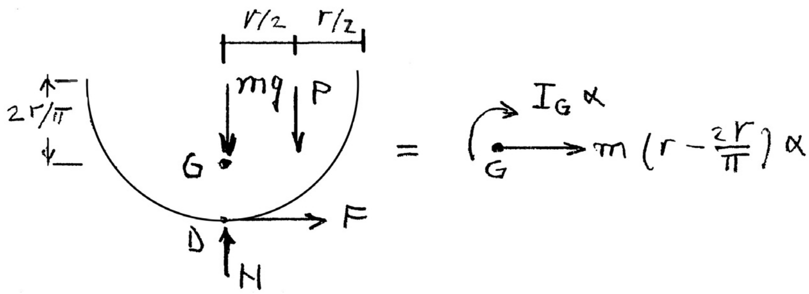
or $(\mu s)_{\min} = 0.333$ ◀



PROBLEM 16.105

A half section of a uniform thin pipe of mass m is at rest when a force $\mathbf{P}$ is applied as shown. Assuming that the section rolls without sliding, determine (*a*) its initial angular acceleration, (*b*) the minimum value of the coefficient of static friction consistent with the motion.

SOLUTION



$$I_G = mr^2 - m\left(\frac{2r}{\pi}\right)^2 = mr^2\left(1 - \frac{4}{\pi^2}\right)$$

$$\xrightarrow{+} \Sigma F_x = F = mr\alpha\left(1 - \frac{2}{\pi}\right)$$

$$+\circlearrowright \Sigma M_D = P\frac{r}{2} = mr^2\left(1 - \frac{4}{\pi^2}\right)\alpha + mr^2\left(1 - \frac{2}{\pi}\right)\left(1 - \frac{2}{\pi}\right)\alpha$$

$$= \left(mr^2 - mr^2\frac{4}{\pi^2}\right)\alpha + \left(mr^2 - 2mr^2\frac{2}{\pi} + mr^2\frac{4}{\pi^2}\right)\alpha$$

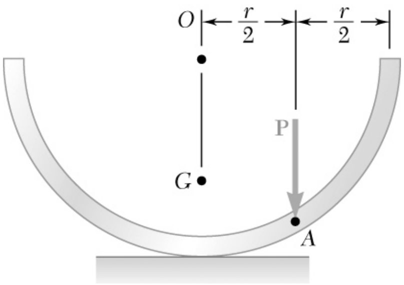
$$= \left(2mr^2 - 2mr^2\frac{2}{\pi}\right)\alpha$$

(*a*) $$\alpha = \frac{P}{4mr\left(1 - \frac{2}{\pi}\right)} \quad \text{or} \quad \alpha = \frac{0.688P}{mr} \blacktriangleleft$$

(*b*) $$F = mr\frac{P\left(1 - \frac{2}{\pi}\right)}{4mr\left(1 - \frac{2}{\pi}\right)} = \frac{P}{4}$$

$$+\uparrow \Sigma F_y = 0: \quad N = mg + P$$

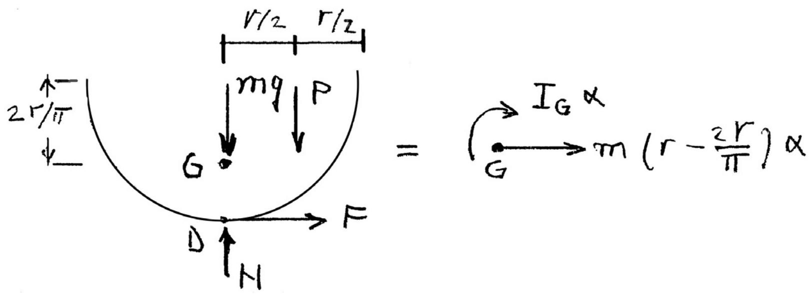
and $$\mu_s = \frac{F}{N} = \frac{P}{4(mg + P)} \quad \text{or} \quad \mu_s = \frac{0.25P}{(mg + P)} \blacktriangleleft$$



PROBLEM 16.105

A half section of a uniform thin pipe of mass m is at rest when a force **P** is applied as shown. Assuming that the section rolls without sliding, determine (*a*) its initial angular acceleration, (*b*) the minimum value of the coefficient of static friction consistent with the motion.

SOLUTION



$$I_G = mr^2 - m\left(\frac{2r}{\pi}\right)^2 = mr^2\left(1-\frac{4}{\pi^2}\right)$$

$$\xrightarrow{+} \Sigma F_x = F = mr\alpha\left(1-\frac{2}{\pi}\right)$$

$$+\circlearrowright \Sigma M_D = P\frac{r}{2} = mr^2\left(1-\frac{4}{\pi^2}\right)\alpha + mr^2\left(1-\frac{2}{\pi}\right)\left(1-\frac{2}{\pi}\right)\alpha$$

$$= \left(mr^2 - mr^2\frac{4}{\pi^2}\right)\alpha + \left(mr^2 - 2mr^2\frac{2}{\pi} + mr^2\frac{4}{\pi^2}\right)\alpha$$

$$= \left(2mr^2 - 2mr^2\frac{2}{\pi}\right)\alpha$$

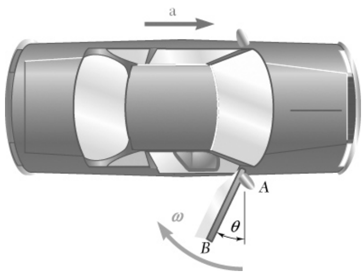
(*a*) $$\alpha = \frac{P}{4mr\left(1-\frac{2}{\pi}\right)}$$ or $$\alpha = \frac{0.688P}{mr}$$ ◀

(*b*) $$F = mr\frac{P\left(1-\frac{2}{\pi}\right)}{4mr\left(1-\frac{2}{\pi}\right)} = \frac{P}{4}$$

$$+\uparrow \Sigma F_y = 0: \quad N = mg + P$$

and $$\mu_s = \frac{F}{N} = \frac{P}{4(mg+P)}$$ or $$\mu_s = \frac{0.25P}{(mg+P)}$$ ◀

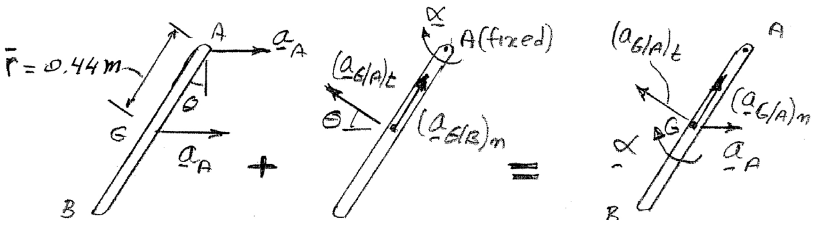
PROBLEM 16.124



A driver starts his car with the door on the passenger's side wide open ($\theta = 0$). The 36-kg door has a centroidal radius of gyration $\bar{k} = 250$ mm, and its mass center is located at a distance $r = 440$ mm from its vertical axis of rotation. Knowing that the driver maintains a constant acceleration of $2\ \text{m/s}^2$, determine the angular velocity of the door as it slams shut ($\theta = 90°$).

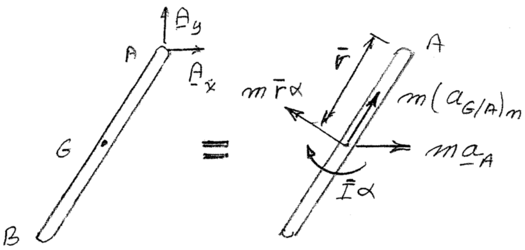
SOLUTION

Kinematics: $\mathbf{a} = a_A$



where $\left(\mathbf{a}_{G/A}\right)_t = \bar{r}\alpha \searrow \theta$

Kinetics:



$$+\circlearrowleft\ \Sigma M_A = \Sigma\left(M_A\right)_{\text{eff}}:\quad 0 = \bar{I}\alpha + \left(m\bar{r}\alpha\right)\bar{r} - ma_A\left(\bar{r}\cos\theta\right)$$

$$m\bar{k}^2\alpha + m\bar{r}^2\alpha = ma_A\bar{r}\cos\theta$$

$$\alpha = \frac{a_A\bar{r}}{\bar{k}^2 + \bar{r}^2}\cos\theta$$

Setting $\alpha = \omega\dfrac{d\omega}{d\theta}$, and using $\bar{r} = 0.44$ m, $\bar{k} = 0.25$ m

$$\omega\frac{d\omega}{d\theta} = \frac{(0.44\ \text{m})a_A}{(0.44\ \text{m})^2 + (0.25\ \text{m})^2}\cos\theta = 1.7181a_A\cos\theta$$

$$\int_0^{\omega_f}\omega d\omega = 1.7181a_A\int_0^{\frac{\pi}{2}}\cos\theta d\theta$$

$$\left|\frac{1}{2}\omega^2\right|_0^{\omega_f} = 1.7181a_A\sin\theta\Big|_0^{\frac{\pi}{2}} \Rightarrow \omega_f^2 = 3.4362a_A \qquad (1)$$

PROBLEM 16.124 CONTINUED

Given $a_A = 2 \text{ m/s}^2$

$$\omega_f^2 = 3.4362(2) = 6.8724 \Rightarrow \omega_f = 2.6215 \text{ rad/s}$$

or $\omega_f = 2.62 \text{ rad/s}$ ↻ ◀