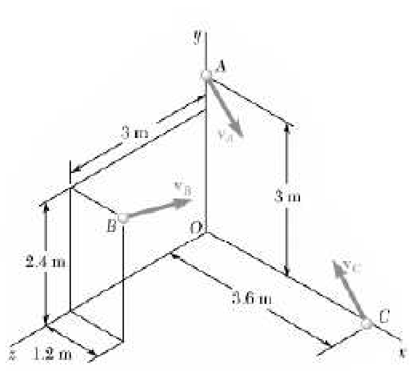
PROBLEM 14.13



A system consists of three particles A, B, and C. We know that $m_A = 3$ kg, $m_B = 2$ kg, and $m_C = 4$ kg and that the velocities of the particles expressed in m/s are, respectively, $\mathbf{v}_A = 4\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$, $\mathbf{v}_B = 4\mathbf{i} + 3\mathbf{j}$, and $\mathbf{v}_C = -2\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$. Determine the angular momentum $\mathbf{H}_O$ of the system about O.

SOLUTION

Linear momentum of each particle expressed in kg·m/s.

$$m_A\mathbf{v}_A = 12\mathbf{i} + 6\mathbf{j} + 6\mathbf{k}$$
$$m_B\mathbf{v}_B = 8\mathbf{i} + 6\mathbf{j}$$
$$m_C\mathbf{v}_C = -8\mathbf{i} + 16\mathbf{j} + 8\mathbf{k}$$

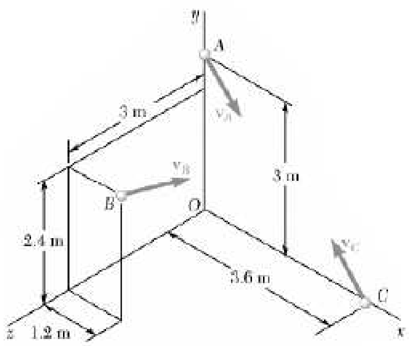
Position vectors, (meters): $\mathbf{r}_A = 3\mathbf{j}$, $\quad \mathbf{r}_B = 1.2\mathbf{i} + 2.4\mathbf{j} + 3\mathbf{k}$, $\quad \mathbf{r}_C = 3.6\mathbf{i}$

Angular momentum about O, $(\text{kg}\cdot\text{m}^2/\text{s})$.

$$\begin{aligned}
\mathbf{H}_O &= \mathbf{r}_A \times (m_A\mathbf{v}_A) + \mathbf{r}_B \times (m_B\mathbf{v}_B) + \mathbf{r}_C \times (m_C\mathbf{v}_C) \\
&= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 3 & 0 \\ 12 & 6 & 6 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1.2 & 2.4 & 3 \\ 8 & 6 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3.6 & 0 & 0 \\ -8 & 16 & 8 \end{vmatrix} \\
&= (18\mathbf{i} - 36\mathbf{k}) + (-18\mathbf{i} + 24\mathbf{j} - 12\mathbf{k}) + (-28.8\mathbf{j} + 57.6\mathbf{k}) \\
&= 0\mathbf{i} - 4.8\mathbf{j} + 9.6\mathbf{k}
\end{aligned}$$

$$\mathbf{H}_O = -\left(4.80\ \text{kg}\cdot\text{m}^2/\text{s}\right)\mathbf{j} + \left(9.60\ \text{kg}\cdot\text{m}^2/\text{s}\right)\mathbf{k} \blacktriangleleft$$

PROBLEM 14.14



For the system of particles of Prob. 14.13, determine (*a*) the position vector $\overline{\mathbf{r}}$ of the mass center G of the system, (*b*) the linear momentum $m\overline{\mathbf{v}}$ of the system, (*c*) the angular momentum $\mathbf{H}_G$ of the system about G. Also verify that he answers to this problem and to problem 14.13 satisfy the equation given in Prob. 14.28.

SOLUTION

Position vectors, (meters): $\mathbf{r}_A = 3\mathbf{j}, \quad \mathbf{r}_B = 1.2\mathbf{i} + 2.4\mathbf{j} + 3\mathbf{k}, \quad \mathbf{r}_C = 3.6\mathbf{i}$

(*a*) Mass center: $(m_A + m_B + m_C)\overline{\mathbf{r}} = m_A\mathbf{r}_A + m_B\mathbf{r}_B + m_C\mathbf{r}_C$

$$9\overline{\mathbf{r}} = (3)(3\mathbf{j}) + (2)(1.2\mathbf{i} + 2.4\mathbf{j} + 3\mathbf{k}) + (4)(3.6\mathbf{i})$$
$$\overline{\mathbf{r}} = 1.86667\mathbf{i} + 1.53333\mathbf{j} + 0.66667\mathbf{k}$$

$$\overline{\mathbf{r}} = (1.867\text{ m})\mathbf{i} + (1.533\text{ m})\mathbf{j} + (0.667\text{ m})\mathbf{k} \blacktriangleleft$$

Linear momentum of each particle, $(\text{kg}\cdot\text{m}^2/\text{s})$.

$$m_A\mathbf{v}_A = 12\mathbf{i} + 6\mathbf{j} + 6\mathbf{k}$$
$$m_B\mathbf{v}_B = 8\mathbf{i} + 6\mathbf{j}$$
$$m_C\mathbf{v}_C = -8\mathbf{i} + 16\mathbf{j} + 8\mathbf{k}$$

(*b*) Linear momentum of the system, $(\text{kg}\cdot\text{m/s.})$

$$m\overline{\mathbf{v}} = m_A\mathbf{v}_A + m_B\mathbf{v}_B + m_C\mathbf{v}_C = 12\mathbf{i} + 28\mathbf{j} + 14\mathbf{k}$$

$$m\overline{\mathbf{v}} = (12.00\text{ kg}\cdot\text{m/s})\mathbf{i} + (28.0\text{ kg}\cdot\text{m/s})\mathbf{j} + (14.00\text{ kg}\cdot\text{m/s})\mathbf{k} \blacktriangleleft$$

Position vectors relative to the mass center, (meters).

$$\mathbf{r}'_A = \mathbf{r}_A - \overline{\mathbf{r}} = -1.86667\mathbf{i} + 1.46667\mathbf{j} - 0.66667\mathbf{k}$$
$$\mathbf{r}'_B = \mathbf{r}_B - \overline{\mathbf{r}} = -0.66667\mathbf{i} + 0.86667\mathbf{j} + 2.33333\mathbf{k}$$
$$\mathbf{r}'_C = \mathbf{r}_C - \overline{\mathbf{r}} = 1.73333\mathbf{i} - 1.53333\mathbf{j} - 0.66667\mathbf{k}$$

PROBLEM 14.14 CONTINUED

(*c*) Angular momentum about G, $\left(\text{kg}\cdot\text{m}^2/\text{s}\right)$.

$$\mathbf{H}_G = \mathbf{r}'_A \times m_A\mathbf{v}_A + \mathbf{r}'_B \times m_B\mathbf{v}_B + \mathbf{r}'_C \times m_C\mathbf{v}_C$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1.86667 & 1.46667 & -0.66667 \\ 12 & 6 & 6 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.66667 & 0.86667 & 2.33333 \\ 8 & 6 & 0 \end{vmatrix}$$

$$+ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1.73333 & -1.53333 & -0.66667 \\ -8 & 16 & 8 \end{vmatrix}$$

$$= \left(12.8\mathbf{i} + 3.2\mathbf{j} - 28.8\mathbf{k}\right) + \left(-14\mathbf{i} + 18.6667\mathbf{j} - 10.9333\mathbf{k}\right)$$

$$+ \left(-1.6\mathbf{i} - 8.5333\mathbf{j} + 15.4667\mathbf{k}\right)$$

$$= -2.8\mathbf{i} + 13.3333\mathbf{j} - 24.2667\mathbf{k}$$

$$\mathbf{H}_G = -\left(2.80\ \text{kg}\cdot\text{m}^2/\text{s}\right)\mathbf{i} + \left(13.33\ \text{kg}\cdot\text{m}^2/\text{s}\right)\mathbf{j} - \left(24.3\ \text{kg}\cdot\text{m}^2/\text{s}\right)\mathbf{k} \blacktriangleleft$$

$$\bar{\mathbf{r}} \times m\bar{\mathbf{v}} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1.86667 & 1.53333 & 0.66667 \\ 12 & 28 & 14 \end{vmatrix}$$

$$= \left(2.8\ \text{kg}\cdot\text{m}^2/\text{s}\right)\mathbf{i} - \left(18.1333\ \text{kg}\cdot\text{m}^2/\text{s}\right)\mathbf{j} + \left(33.8667\ \text{kg}\cdot\text{m}^2/\text{s}\right)\mathbf{k}$$

$$\mathbf{H}_G + \bar{\mathbf{r}} \times m\bar{\mathbf{v}} = -\left(4.8\ \text{kg}\cdot\text{m}^2/\text{s}\right)\mathbf{j} + \left(9.6\ \text{kg}\cdot\text{m}^2/\text{s}\right)\mathbf{k}$$

Angular momentum about O.

$$\mathbf{H}_O = \mathbf{r}_A \times \left(m_A\mathbf{v}_A\right) + \mathbf{r}_B \times \left(m_B\mathbf{v}_B\right) + \mathbf{r}_C \times \left(m_C\mathbf{v}_C\right)$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 3 & 0 \\ 12 & 6 & 6 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1.2 & 2.4 & 3 \\ 8 & 6 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3.6 & 0 & 0 \\ -8 & 16 & 8 \end{vmatrix}$$

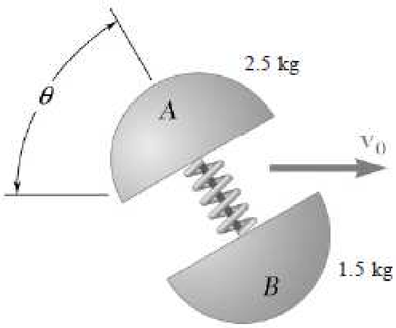
$$= \left(18\mathbf{i} - 36\mathbf{k}\right) + \left(-18\mathbf{i} + 24\mathbf{j} - 12\mathbf{k}\right) + \left(-28.8\mathbf{j} + 57.6\mathbf{k}\right)$$

$$= -\left(4.8\ \text{kg}\cdot\text{m}^2/\text{s}\right)\mathbf{j} + \left(9.6\ \text{kg}\cdot\text{m}^2/\text{s}\right)\mathbf{k}$$

Note that

$$\mathbf{H}_O = \mathbf{H}_G + \bar{\mathbf{r}} \times m\bar{\mathbf{v}}$$

PROBLEM 14.39

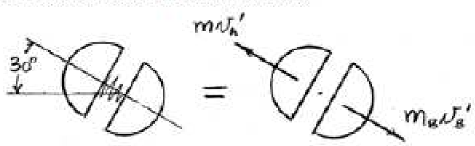


Two hemispheres are held together by a cord which maintains a spring under compression (the spring is not attached to the hemispheres). The potential energy of the compressed spring is 120 N·m and the assembly has an initial velocity $\mathbf{v}_0$ of magnitude $v_0 = 8$ m/s. Knowing that the cord is severed when $\theta = 30°$, causing the hemispheres to fly apart, determine the resulting velocity of each hemisphere.

SOLUTION

Use a frame of reference moving with the mass center.

Conservation of momentum:



$$0 = -m_A v_A' + m_B v_B'$$

$$v_A' = \frac{m_B}{m_A} v_B'$$

Conservation of energy:

$$V = \frac{1}{2} m_A (v_A')^2 + \frac{1}{2} m_B (v_B')^2 = \frac{1}{2} m_A \left(\frac{m_B}{m_A} v_B' \right)^2 + \frac{1}{2} m_B (v_B')^2$$

$$= \frac{m_B (m_A + m_B)}{2 m_A} (v_B')^2$$

$$v_B' = \sqrt{\frac{2 m_A V}{m_B (m_A + m_B)}}$$

Data: $m_A = 2.5$ kg $\quad m_B = 1.5$ kg

$V = 120$ N.m

$$v_B' = \sqrt{\frac{(2)(2.5)(120)}{(1.5)(4)}} = 10 \text{ m/s} \quad v_B' = 10 \text{ m/s} \ \nwarrow 30°$$

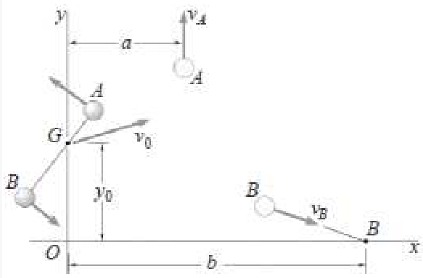
$$v_A' = \frac{1.5}{2.5}(10) = 6.0 \text{ m/s} \quad v_A' = 6.0 \text{ m/s} \ \searrow 30°$$

Velocities of A and B:

$\mathbf{v}_A = [8 \text{ m/s} \longrightarrow] + [6 \text{ m/s} \searrow]$ $\qquad \mathbf{v}_A = 4.1$ m/s $\nearrow$ 46.94° ◀

$\mathbf{v}_B = [8 \text{ m/s} \longrightarrow] + [10 \text{ m/s} \nwarrow]$ $\qquad \mathbf{v}_B = 17.40$ m/s $\nwarrow$ 16.69° ◀

PROBLEM 14.57



Two small disks, A and B, of mass 2.4 kg and 1.2 kg, respectively, can slide on a horizontal, frictionless surface. They are connected by a cord, 900 mm long, and spin counterclockwise about their mass center G at a rate of 8 rad/s. At $t = 0$, the coordinates of G are $\bar{x}_0 = 0$, $\bar{y}_0 = 2.48$ m, and its velocity is $\bar{\mathbf{v}}_0 = (1.92 \text{ m/s})\mathbf{i} + (0.48 \text{ m/s})\mathbf{j}$. Shortly thereafter the cord breaks; disk A is then observed to move along a path parallel to the y axis and disk B along a path which intersects the x axis at a distance $b = 8$ m from O. Determine (a) the velocities of A and B after the cord breaks, (b) the distance a from the y axis to the path of A.

SOLUTION

Conservation of linear momentum: $$(m_A + m_B)\bar{\mathbf{v}}_0 = m_A\mathbf{v}_A + m_B\mathbf{v}_B$$

$$3.6(1.92\mathbf{i} + 0.48\mathbf{j}) = 2.4\, v_A\mathbf{j} + 1.2(v_B)_x\,\mathbf{i} + 1.2(v_B)_y\,\mathbf{j}$$

$$\mathbf{i}:\ 6.912 = 1.2(v_B)_x \qquad (v_B)_x = 5.76 \text{ m/s}$$

$$\mathbf{j}:\ 1.728 = 2.4v_A - 1.2(v_B)_y \qquad (v_B)_y = 1.44 - 2v_A$$

Speeds relative to the mass center: $$u_A = \frac{1}{3}(l\omega) = \frac{1}{3}(0.900)(8) = 2.4 \text{ m/s}$$

$$u_B = \frac{2}{3}(l\omega) = \frac{2}{3}(0.900)(8) = 4.8 \text{ m/s}$$

Initial kinetic energy: $$T_1 = \frac{1}{2}(m_A + m_B)\left[(v_0)_x^2 + (v_0)_y^2\right] + \frac{1}{2}m_A u_A^2 + \frac{1}{2}m_B u_B^2$$

$$T_1 = \frac{1}{2}(3.6)\left(1.92^2 + 0.48^2\right) + \frac{1}{2}(2.4)(2.4)^2 + \frac{1}{2}(1.2)(4.8)^2 = 27.78624 \text{ kg}\cdot\text{m}^2/\text{s}^2$$

Final kinetic energy: $$T_2 = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B(v_B)_x^2 + \frac{1}{2}m_B(v_B)_y^2$$

$$T_2 = \frac{1}{2}(2.4)v_A^2 + \frac{1}{2}(1.2)(5.76)^2 + \frac{1}{2}(1.2)(1.44 - 2v_A)^2$$

$$= 3.6v_A^2 - 3.456v_A + 21.15072$$

Conservation of energy: $$T_1 = T_2$$

(a) $$3.6v_A^2 - 3.456v_A - 6.63552 = 0, \qquad v_A = 1.92 \text{ m/s}$$

$\mathbf{v}_A = 1.920 \text{ m/s} \uparrow$ ◀

$$(v_B)_y = 1.44 - (2)(1.92) = -2.4 \text{ m/s} \qquad \mathbf{v}_B = (5.76 \text{ m/s})\mathbf{i} - (2.4 \text{ m/s})\mathbf{j}$$

$\mathbf{v}_B = 6.24 \text{ m/s} \searrow 22.6°$ ◀

PROBLEM 14.57 CONTINUED

Conservation of angular momentum about O:

$$(H_O)_1 = -\bar{y}_0(m_A + m_B)(v_0)_x + H_G = -\bar{y}_0(m_A + m_B)(v_0)_x + m_A\frac{l}{3}u_A + m_B\frac{2l}{3}u_B$$

$$= -(2.48)(3.6)(1.92) + (2.4)(0.3)(2.4) + (1.2)(0.6)(4.8) = -11.95776 \text{ kg}\cdot\text{m}^2\text{/s}$$

$$(H_O)_2 = m_A v_A a + m_B(v_B)_y b = (2.4)(1.92)a + (1.2)(-2.4)(8)$$

$$= 4.608a - 23.04$$

$$(H_O)_2 = (H_O)_1 \qquad 4.608a - 23.04 = -11.95776$$

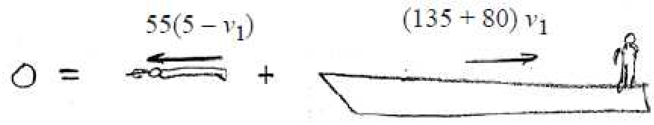
(*b*) $a = 2.405 \text{ m}$ $\qquad a = 2.41 \text{ m}$ ◀

PROBLEM 14.108

A 80 kg man and a 55 kg woman stand at opposite ends of a 135 kg boat, ready to dive, each with a 5 m/s velocity relative to the boat. Determine the velocity of the boat after they have both dived, if (*a*) the woman dives first, (*b*) the man dives first.

SOLUTION

(*a*) *Woman dives first.*

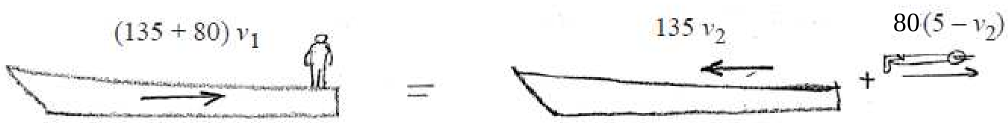


Conservation of momentum:

$$55\,(5 - v_1) - (135 + 80)\quad v_1 = 0$$

$$v_1 = \frac{(55)(5)}{270} = 1.019 \text{ m/s} \longrightarrow$$

Man dives next. Conservation of momentum:



$$-(135 + 80)v_1 = -135\,v_2 + 80\,(5 - v_2)$$

$$v_2 = \frac{-215v_1 + (80)(5)}{215} = 0.8415 \text{ m/s}$$

$\mathbf{v}_2 = 0.84$ m/s $\longleftarrow$ ◀

(*b*) *Man dives first.*

Conservation of momentum:

$$80\,(5 - v_1') - (135 + 55)\,v_1' = 0$$

$$v_1' = \frac{(80)(5)}{270} = 1.481 \text{ m/s} \longleftarrow$$

Woman dives next. Conservation of momentum:

$$-(135 + 55)\,v_1' = \;135\,v_2' \quad 55\,(5 - v_2')$$

$$v_2' = \frac{-190v_1' + (55)(5)}{190} = -0.0336 \text{ m/s}$$

$\mathbf{v}_2' = 0.0336$ m/s $\leftarrow$ ◀