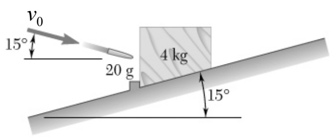
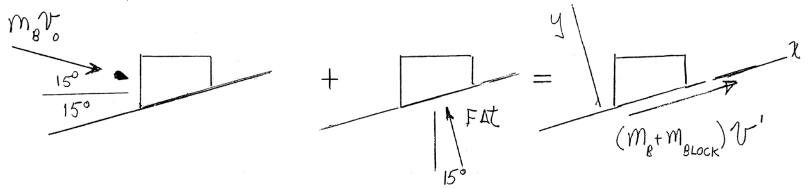


PROBLEM 13.148

A 20-g bullet is fired into a 4-kg wooden block and becomes embedded in it. Knowing that the block and bullet then move up the smooth incline for 1.2 s before they come to a stop, determine (*a*) the magnitude of the initial velocity of the bullet, (*b*) the magnitude of the impulse of the force exerted by the bullet on the block.

SOLUTION

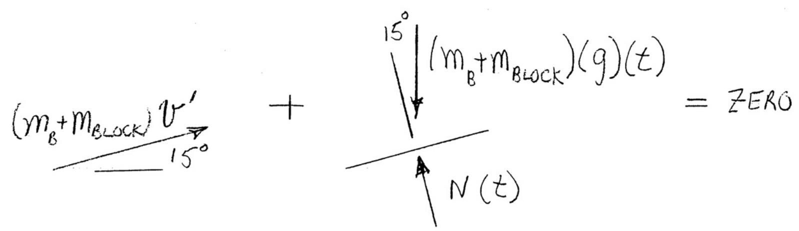
Initial impact (Bullet + Block)



$$\overset{+}{\rightarrow}\ x: \quad m_B v_0 \cos 30° + 0 = (m_B + m_{\text{block}})v' \qquad (1)$$

$$+\uparrow\ y: \quad -m_B v_0 \sin 30° + F\Delta t = 0 \qquad (2)$$

After impact



$$x: \quad 4.02v' - 4.02(9.81)(1.2)\sin 15° = 0$$

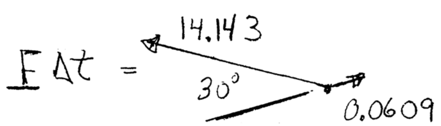
$$v' = 3.0468 \text{ m/s}$$

From (1)
$$v_0 = \frac{4.02v'}{0.02\cos 30},$$

(*a*) $v_0 = 707$ m/s ◀

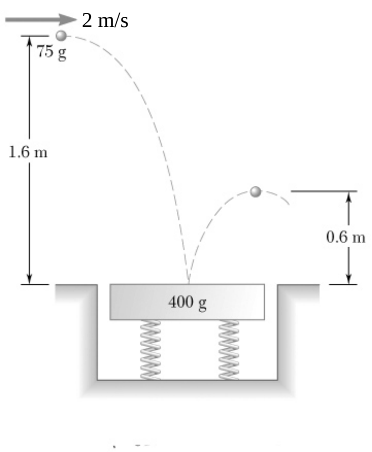
Bullet alone

$$m_B v_0 = 0.02(707.15) = 14.143; \qquad m_B v' = 0.02(3.0468) = 0.0609$$



$|F\Delta t| = 14.09$ N·s ◀

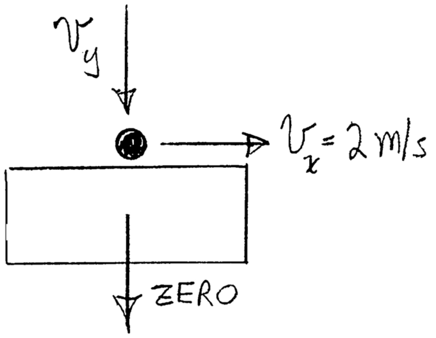
PROBLEM 13.151



A 75-g ball is projected from a height of 1.6 m with a horizontal velocity of 2 m/s and bounces from a 400-g smooth plate supported by springs. Knowing that the height of the rebound is 0.6 m, determine (*a*) the velocity of the plate immediately after the impact, (*b*) the energy lost due to the impact.

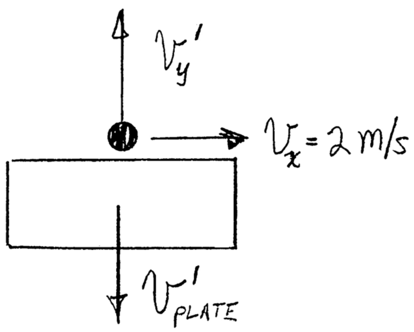
SOLUTION

Just before impact



$$v_y = \sqrt{2g(1.6)} = 5.603 \text{ m/s}$$

Just after impact



$$v_y = \sqrt{2g(0.6)} = 3.431 \text{ m/s}$$

(*a*) Conservation of momentum: $(+y\downarrow)$

$$m_{\text{ball}} v_y + 0 = -m_{\text{ball}} v'_y + m_{\text{plate}} v'_{\text{plate}}$$

$$(0.075)(5.603) + 0 = -0.075(3.431) + 0.4 v'_{\text{plate}}$$

$$v'_{\text{plate}} = 1.694 \text{ m/s} \downarrow \blacktriangleleft$$

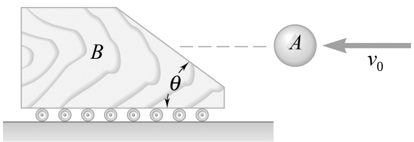
(*b*) Energy loss

Initial energy $\quad (T+V)_1 = \frac{1}{2}(0.075)(2)^2 + 0.075g(1.6)$

Final energy $\quad (T+V)_2 = \frac{1}{2}(0.075)(2)^2 + 0.075g(0.6) + \frac{1}{2}(0.4)(1.694)^2$

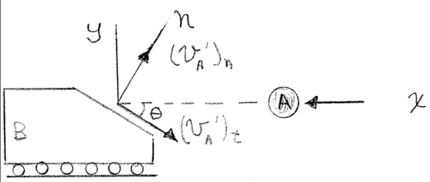
$$\text{Energy lost} = (1.3272 - 1.1653)\text{J} = 0.1619\text{J} \blacktriangleleft$$

PROBLEM 13.169

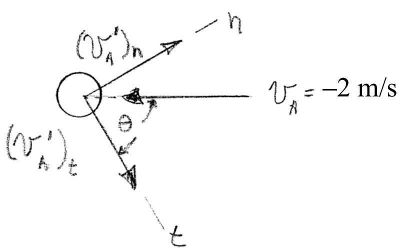


A 1.5 kg sphere A moving with a velocity $\mathbf{v}_0$ parallel to the ground and of magnitude $v_0 = 2$ m/s strikes the inclined face of a 6 kg wedge B, which can roll freely on the ground and is initially at rest. Knowing that $\theta = 60^\circ$ and that the coefficient of restitution between the sphere and the wedge is $e = 1$, determine the velocity of the wedge immediately after impact.

SOLUTION



$$v_0 = v_A = -2 \text{ m/s}, \ \theta = 60^\circ$$



A alone momentum conserved in t-direction

$$m_A v_A \cos 60^\circ = m_A (v'_A)_t$$

$$(v'_A)_t = -1 \text{ m/s}$$

A and B total momentum conserved along the x-axis

$$m_A v_A + m_B v_B = m_A\left[(v'_A)_t \cos 60^\circ + (v'_A)_n \sin 60^\circ\right] - m_B v'_B$$

$$1.5 \text{ kg } (-2 \text{ m/s}) + 0 = 1.5 \text{ kg } [(-1 \text{ m/s}) \cos 60^\circ + (v'_A)_n \sin 60^\circ] - (6 \text{ kg})\ v'_B$$

$$-3 = -0.75 + 1.3\ (v'_A)_n - 6 v'_B$$

$$-2.25 = 1.3\ (v'_A)_n - 6 v'_B \qquad (1)$$

Relative velocities in the n direction

$$\left[v_A \sin 60^\circ - (v_B)_n\right] e = (v'_B)_n - (v'_A)_n$$

$$(-2) \sin 60^\circ (1) = -v'_B \sin 60^\circ - (v'_A)_n$$

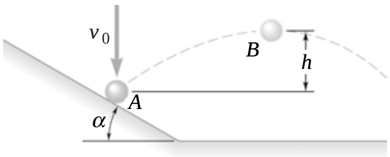
$$-1.73 = -0.866 v'_B - (v'_A)_n \qquad (2)$$

Solving (1) and (2) simultaneously

$$(v'_A)_n = 1.183 \text{ m/s}$$

$$v'_B = 0.631 \text{ m/s}$$

$$v'_B = 0.63 \text{ m/s} \ \longleftarrow \ \blacktriangleleft$$

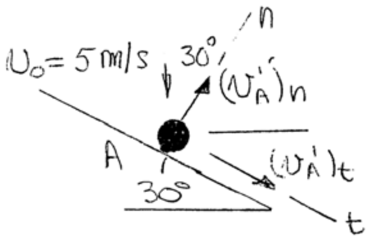


PROBLEM 13.173

A sphere rebounds as shown after striking an inclined plane with a vertical velocity $\mathbf{v}_0$ of magnitude $v_0 = 5$ m/s. Knowing that $\alpha = 30°$ and $e = 0.8$ between the sphere and the plane, determine the height h reached by the sphere.

SOLUTION

Rebound at A



Conservation of momentum – t-direction

$$mv_0 \sin 30° = m(v_A')_t \Rightarrow (v_A')_t = 5 \sin 30°$$

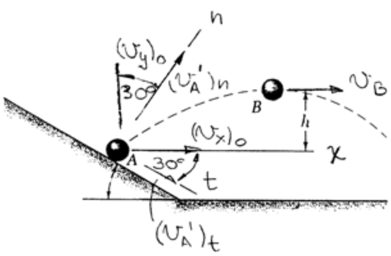
$$(v_A')_t = 2.5 \text{ m/s}$$

Relative velocities in the n-direction

$$(-v_0 \cos 30° - 0)e = 0 - (v_A')_n \Rightarrow (v_A')_n = (5 \cos 30)(0.8)$$

$$(v_A')_n = 3.464 \text{ m/s}$$

Projectile motion between A and B



After rebound $\qquad (v_x)_0 = (v_A')_t \cos 30° + (v_A')_n \sin 30°$

$$= 2.5 \cos 30° + 3.464 \sin 30°$$

$$(v_x)_0 = 3.897 \text{ m/s}$$

$$(v_y)_0 = -(v_A')_t \sin 30° + (v_A')_n \cos 30°$$

$$= -2.5 \sin 30° + 3.464 \cos 30°$$

$$(v_y)_0 = 1.75 \text{ m/s}$$

x direction: $\qquad x = (v_x)_0 t = 3.897t, \; v_x = (v_x)_0 = 3.897 \text{ m/s}$

y direction: $\qquad y = (v_y)_0 t - \frac{1}{2}gt^2 = 1.75t - 4.905t^2$

$$v_y = (v_y)_0 - gt = 1.75 - 9.81t$$

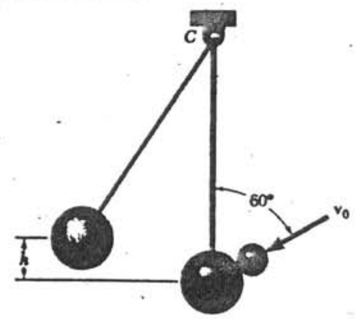
At B: $\qquad v_y = 0 = 1.75 - 9.81t_{A-B} \Rightarrow t_{A-B} = 0.17839 \text{ s}$

$$y = h = (v_y)_0 t_{A-B} - 4.905t_{A-B}^2 = 1.75(0.17839) - 4.905(0.17839)^2$$

$$h = 0.15609 \text{ m}$$

$h = 156.1$ mm ◀

13.189



GIVEN:

$m_B = 340$ g

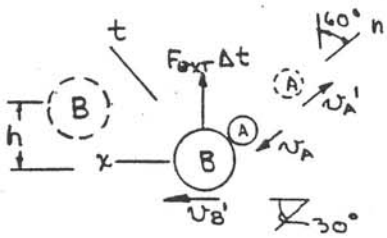
$m_A = 170$ g

$v_0 = 1.5$ m/s AT 60° AS SHOWN

$e = 1$

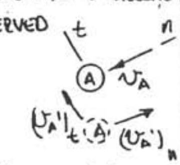
FIND:

HEIGHT h REACHED BY BALL B



BALL A ALONE

MOMENTUM IN t-DIRECTION CONSERVED



$m_A(v_A)_t = m_A(v_A')_t$

$(v_A)_t = 0 = (v_A')_t$

THUS $(v_A')_n = v_A'$

TOTAL MOMENTUM IN THE X-DIRECTION IS CONSERVED

$$m_A v_A \sin 60° + m_B (v_B)_x = m_A(-v_A')\sin 60 + m_B v_B'$$

$v_A = v_0 = 1.5$ m/s $\quad (v_B)_x = 0$

$$(0.17)(1.5)(\sin 60°) + 0 = -(0.17)(v_A')(\sin 60°) + (0.34)v_B'$$

$$0.2208 = -0.1472\, v_A' + 0.34\, v_B' \quad (1)$$

RELATIVE VELOCITY IN THE n-DIRECTION

$$[-v_A - (v_B)_n]\, e = -v_B' \cos 30° - v_A'$$

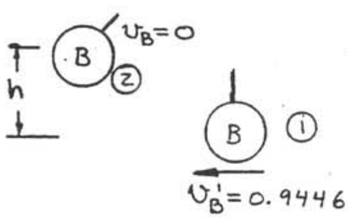
$$(-1.5 - 0)(1) = -0.866\, v_B' - v_A' \quad (2)$$

SOLVE EQ (1) AND (2) SIMULTANEOUSLY

$$v_B' = 0.9446 \text{ m/s} \qquad v_A' = 0.6820 \text{ m/s}$$

CONSERVATION OF ENERGY

BALL B



$T_1 = \frac{1}{2} m_B (v_B')^2$

$T_1 = \frac{1}{2}\frac{W_B}{g}(0.9446)^2$

$V_1 = 0$

$T_2 = 0 \quad V_2 = W_B h$

$T_1 + V_1 = T_2 + V_2 \quad \frac{1}{2}\frac{W_B}{g}(0.9446)^2 = 0 + W_B h$

$$h = \frac{(0.9446)^2}{(2)(9.81)} = 0.0455 \text{ m}$$

$h = 45.5$ mm ◀