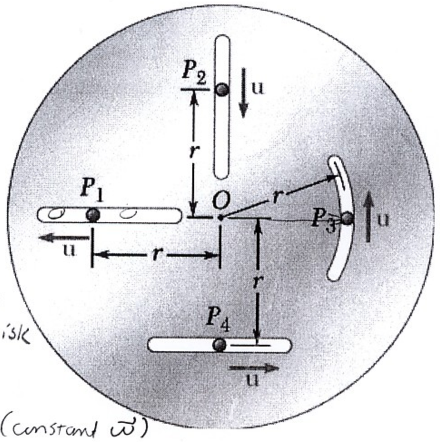
Solution to Assignment 6
CIVE 281

PROBLEM 15.156

Four pins slide in four separate slots cut in a circular plate as shown. When the plate is at rest, each pin has a velocity directed as shown and of the same constant magnitude u. If each pin maintains the same velocity relative to the plate when the plate rotates about O with a constant counterclockwise angular velocity ω, determine the acceleration of each pin.



Frame of reference placed @ position "O". ∴ Axyz is centered @ center of disk

For all pins: $\dot{\vec{\omega}} = 0$ (constant $\vec{\omega}$)

$\vec{a}_A = 0$ (no translation of ref. frame ... origines of fixed frame Oxyz + reference frame Axyz correspond)

for pin 1: $$\vec{a}_{P_1} = \cancelto{0}{\vec{a}_A} + \cancelto{0}{\dot{\vec{\omega}}} \times \cancelto{-r\hat{i}}{\vec{r}_{P/A}} + \vec{\omega} \times (\vec{\omega} \times \cancelto{-r\hat{i}}{\vec{r}_{P/A}}) + 2\vec{\omega} \times \cancelto{-u\hat{i}}{\dot{\vec{r}}_{P/A}} + \cancelto{0}{\ddot{\vec{r}}_{P/A}}$$

where $\vec{\omega} = (0\ 0\ \omega)$

$$\boxed{\vec{a}_{P_1} = r\omega^2\hat{i} - 2u\omega\hat{j}}$$

for pin 2: $$\vec{a}_{P_2} = \cancelto{0}{\vec{a}_A} + \cancelto{0}{\dot{\vec{\omega}}} \times \cancelto{-r\hat{j}}{\vec{r}_{P/A}} + \vec{\omega} \times (\vec{\omega} \times \cancelto{r\hat{j}}{\vec{r}_{P/A}}) + 2\vec{\omega} \times \cancelto{-u\hat{j}}{\dot{\vec{r}}_{P/A}} + \cancelto{0}{\ddot{\vec{r}}_{P/A}}$$

$$\boxed{\vec{a}_{P_2} = 2u\omega\hat{i} - r\omega^2\hat{j}}$$

for pin 4: $$\vec{a}_{P_4} = \cancelto{0}{\vec{a}_A} + \cancelto{0}{\dot{\vec{\omega}}} \times \cancelto{-r\hat{j}}{\vec{r}_{P/A}} + \vec{\omega} \times (\vec{\omega} \times \cancelto{-r\hat{j}}{\vec{r}_{P/A}}) + 2\vec{\omega} \times \cancelto{u\hat{i}}{\dot{\vec{r}}_{P/A}} + \cancelto{0}{\ddot{\vec{r}}_{P/A}}$$

$$\boxed{\vec{a}_{P_4} = (r\omega^2 + 2u\omega)\hat{j}}$$

15.156 cont'd)

for pin 3 :

$$\vec{a}_{P_3} = \overset{\phi}{\cancel{a_A}} + \overset{\phi}{\cancel{\dot{\vec{\omega}}}} \times \overset{r\hat{i}}{\cancel{\vec{r}_{P/A}}} + \vec{\omega} \times (\vec{\omega} \times \overset{r\hat{i}}{\cancel{\vec{r}_{P/A}}}) + 2\vec{\omega} \times \overset{u\hat{j}}{\cancel{\dot{\vec{r}}_{P/A}}} + \ddot{\vec{r}}_{P/A}$$

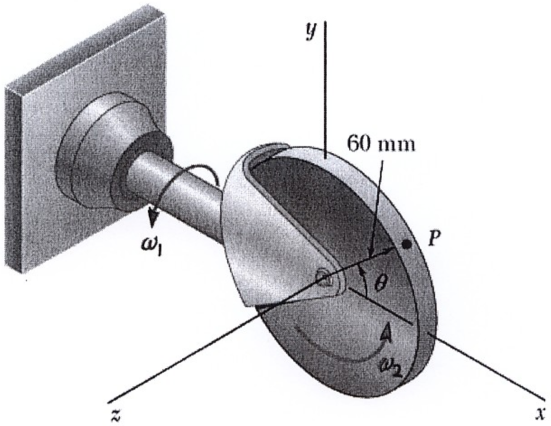
$\ddot{\vec{r}}_{P/A}$ has a normal acceleration component because of the curved path.

$$\ddot{\vec{r}}_{P/A} \Rightarrow \frac{du}{dt} = \frac{d^2r}{dt^2} = u\frac{du}{dr} = \frac{-u^2}{r} \Rightarrow \frac{-u^2}{r}\hat{i}$$

$$\boxed{\vec{a}_{P_3} = -(r\omega^2 + 2u\omega + \frac{u^2}{r})\hat{i}}$$

PROBLEM 15.190

A 60-mm-radius disk spins at the constant rate $\omega_2 = 4$ rad/s about an axis held by a housing attached to a horizontal rod that rotates at the constant rate $\omega_1 = 5$ rad/s. For the position shown, determine (*a*) the angular acceleration of the disk, (*b*) the acceleration of point *P* on the rim of the disk if $\theta = 0$, (*c*) the acceleration of point *P* on the rim of the disk if $\theta = 90°$.



NOTE:

Two different locations of the frame of reference are used to solve part (a) & parts (b) and (c).

(a) Reference frame is placed on the clamp (@ the center of xyz axis in the schematic). Thus it is not placed on the disk + ∴ does NOT rotate with $\vec{\omega}_2$ but does rotate with $\vec{\omega}_1$.

This is done to facilitate the solution to the following equation:

$$\dot{\vec{Q}}_{OXYZ} = \dot{\vec{Q}}_{Axyz} + \vec{\omega} \times \vec{Q}$$

Now solve for $\vec{\alpha}$:

$\vec{\alpha} = \dot{\vec{\omega}} = \dot{\vec{\omega}}_1 + \dot{\vec{\omega}}_2$ but since Axyz is on the clamp it rotates w/ $\vec{\omega}_1$ & ∴ $\dot{\vec{\omega}}_1 = 0$.

Thus $\vec{\alpha} = \dot{\vec{\omega}}_2$

Now plug in $\vec{\omega}_2$ as $\vec{Q}$:

$$\dot{\vec{\omega}}_{2\,OXYZ} = \dot{\vec{Q}}_{Axyz} + \underset{\text{of reference frame... ∴ ang. velocity of } Axyz = \vec{\omega}_1}{\vec{\omega}} \times \vec{Q}$$

$$\dot{\vec{\omega}}_2 = \underset{\text{w.r.t. reference frame... } \vec{\omega}_2 = \text{constant ∴ } \dot{\vec{\omega}}_2 = 0}{\overset{\nearrow 0}{\cancel{\dot{\vec{\omega}}_2}}} + \vec{\omega}_1 \times \underset{\text{parameter of interest}}{\vec{\omega}_2}$$

$$\dot{\vec{\omega}}_2 = \vec{\omega}_1 \times \vec{\omega}_2$$

$$\dot{\vec{\omega}}_2 = \vec{\alpha} = \dot{\vec{\omega}} = \boxed{-20\hat{j} \text{ rad/s}}$$

(b) + (c)

Reference frame is placed @ center of disk (thus Axyz rotates with $\vec{\omega}_1$ + $\vec{\omega}_2$).

$(5\ 0\ 4) = \omega_1 + \omega_2 = $ ang. vel. of ref frame Axyz

$$\vec{a}_P = \overset{\nearrow 0}{\cancel{\vec{a}_A}} + \overset{\nearrow -20\hat{j}}{\cancel{\dot{\vec{\omega}}}} \times \vec{r}_{P/A} + \overset{\nearrow}{\cancel{\vec{\omega}}} \times (\overset{\uparrow}{\cancel{\vec{\omega}}} \times \vec{r}_{P/A}) + 2\underset{\swarrow (5\ 0\ 4)}{\cancel{\vec{\omega}}} \times \underset{\downarrow 0}{\cancel{\dot{\vec{r}}_{P/A}}} + \underset{\downarrow 0}{\cancel{\ddot{\vec{r}}_{P/A}}}$$

(both $\dot{\vec{r}}_{P/A}$ + $\ddot{\vec{r}}_{P/A}$ are 0 because the point P does not move independant of the ref frame Axyz... i.e. The ref frame moves with both $\vec{\omega}_1$ + $\vec{\omega}_2$ + P does not move independ of these rotations thus $\dot{\vec{r}} + \ddot{\vec{r}} = 0$)

part (b) $\theta = 0°$

∴ $r = (60\ \ 0\ \ 0)$ mm

& $\boxed{\vec{a}_P = -960\hat{i} + 2400\hat{k} \text{ mm/s}}$

part (c) $\theta = 90°$

∴ $r = (0\ \ 60\ \ 0)$ mm

& $\boxed{\vec{a}_P = -2460\hat{j} \text{ mm/s}}$

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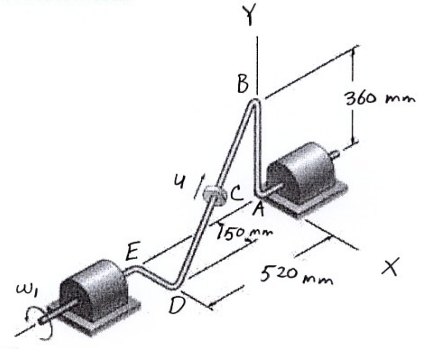
PROBLEM 15.215

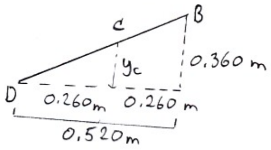
The bent rod shown rotates at the constant rate $\omega_1 = 5$ rad/s and collar *C* moves toward point *B* at a constant relative speed $u = 975$ mm/s. Knowing that collar *C* is halfway between points *B* and *D* at the instant shown, determine its velocity and acceleration.



Place reference frame at point A on schematic.

Need to solve for $\vec{r}_{C/A}$ + $\dot{\vec{r}}_{C/A}$ in order to solve the velocity + acceleration equations.

$\vec{r}_{C/A}$:



$$\frac{0.360}{0.520} = \frac{y_c}{0.260} \; ; \; y_c = 0.180 \text{ m}$$

$$\therefore \; \vec{r}_{C/A} = (0.075\,\hat{i} + 0.180\,\hat{j} + 0.260\,\hat{k})\,\text{m}$$

$\dot{\vec{r}}_{C/A}$:

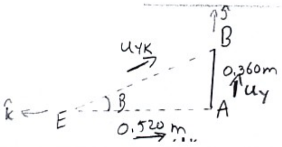
$$\overline{BO} = \sqrt{(0.150)^2 + (0.360)^2 + (0.520)^2}$$

$$= 0.650 \text{ m}$$

$$\alpha = 13.34°$$

$$* \; \vec{u}_x = -(0.975 \sin(13.34)) = -0.225\,\hat{i} \text{ m/s}$$

$$\vec{u}_{YK} = (0.975 \cos(13.34)) = 0.9487 \text{ m/s (direction in schematic)}$$



$$\beta = 34.69°$$

$$* \; \vec{u}_y = \sin(34.69) \times 0.9487 = 0.540\,\hat{j} \text{ m/s}$$

$$* \; \vec{u}_k = -\cos(34.69) \times 0.9487 = -0.778\,\hat{k} \text{ m/s}$$

$$\therefore \ \dot{\vec{r}}_{C/A} = \vec{u}_x + \vec{u}_y + \vec{u}_k$$

$$= (-0.225\,\hat{i} + 0.540\,\hat{j} - 0.778\,\hat{k})\ m/s$$

Now use the velocity equation:

$$\vec{v}_C = \overset{\phi}{\cancel{\vec{v}_A}} + \vec{\omega} \times \overset{(0.075\ 0.180\ 0.260)}{\vec{r}_{C/A}} + \overset{(-0.225\ 0.540\ -0.778)}{\dot{\vec{r}}_{C/A}}$$

$L_D = \vec{\omega}_1$ = ang vel of ref frame = (0 0 5)

$$\vec{v}_C = \boxed{-1.125\,\hat{i} + 0.915\,\hat{j} - 0.778\,\hat{k}}$$

Now use the acceleration equation:

$$\vec{a}_C = \overset{\phi}{\cancel{\vec{a}_A}} + \overset{\phi\ (\vec{\omega}_1 = \text{CONSTANT})}{\cancel{\dot{\vec{\omega}}_1}} \times \vec{r}_{C/A} + \vec{\omega}_1 \times (\vec{\omega}_1 \times \vec{r}_{C/A}) + 2\vec{\omega}_1 \times \dot{\vec{r}}_{C/A} + \overset{\phi\ (\dot{\vec{r}}_{C/A} = \text{CONSTANT} = u)}{\cancel{\ddot{\vec{r}}_{C/A}}}$$

$$\vec{a}_C = \boxed{-7.28\,\hat{i} - 6.75\,\hat{j}}\ m/s$$

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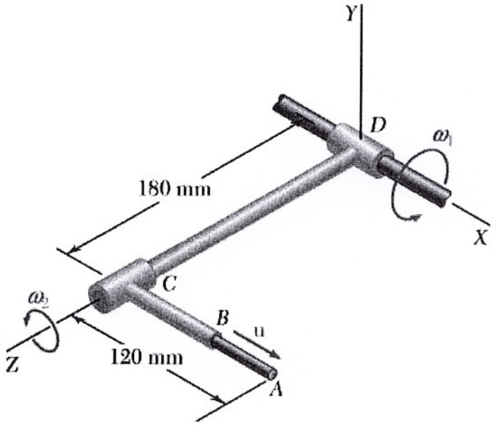
PROBLEM 15.235

In the position shown the thin rod moves at a constant speed $u = 60$ mm/s out of the tube BC. At the same time tube BC rotates at the constant rate $\omega\ 2 = 1.5$ rad/s with respect to arm CD. Knowing that the entire assembly rotates about the X axis at the constant rate $\omega\ 1 = 1.2$ rad/s, determine the velocity and acceleration of end A of the rod.



Velocity :

Frame of reference centered at point D, on rigid body DCB.

$$\therefore \vec{\omega} = \vec{\omega}_1 + \vec{\omega}_2 = 1.2\hat{i} + 1.5\hat{k} \text{ rad/s}$$

$$\vec{v}_A = \underset{\text{Translation of ref frame}}{\vec{v}_D} + \vec{v}_{A/D} + \dot{\vec{r}}_{A/D}$$

$$= \cancelto{0}{\vec{v}_D} + \vec{\omega} \times \vec{r}_{A/D} + \dot{\vec{r}}_{A/D}$$

$$= \overset{\text{rad/s}}{(1.2\hat{i} + 1.5\hat{k})} \times \overset{\text{m/s}}{(0.120\hat{i} + 0.180\hat{k})} + \dot{\vec{r}}_{A/D}$$

Aside $\dot{\vec{r}}_{A/D}$ = vel of A relative to the ref. frame.

$$\dot{\vec{r}}_{A/D} = u\hat{i} = 0.06 \text{ m/s } \hat{i}$$

$$\therefore \vec{v}_A = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1.2 & 0 & 1.5 \\ 0.120 & 0 & 0.180 \end{vmatrix} + (0.06\ \ 0\ \ 0) = \boxed{0.06\,\hat{i} - 0.036\,\hat{j} \text{ m/s}}$$

15.235 cont'd)

acceleration:

New reference frame: place reference frame at pt D such that it does not rotate with ω_2 but only rotates with ω_1. (Same as problem 15.190)

$\vec{\alpha} = \dot{\vec{\omega}}_1 + \dot{\vec{\omega}}_2$

since the ref frame rotates w/ ω_1 then $\dot{\vec{\omega}}_1 = \phi$.

$$\therefore \quad \dot{\vec{\omega}} = \dot{\vec{\omega}}_2$$

now use: $\dot{\vec{Q}}_{OXYZ} = \overset{\text{w.r.t. ref frame}}{\dot{\vec{Q}}_{Axyz}} + \underset{\text{angular velocity of ref. frame}}{\vec{\omega}} \times \overset{\text{vector Q w.r.t. OXYZ (fixed frame)}}{\vec{Q}}$

where $\vec{Q} = \vec{\omega}_2$

$$\dot{\vec{\omega}}_2 = \cancelto{\phi}{\dot{\vec{\omega}}_2} + (\vec{\omega}_1 \times \vec{\omega}_2)$$

$$\dot{\vec{\omega}}_2 = (1.2 \;\; 0 \;\; 0) \times (0 \;\; 0 \;\; 1.5)$$

$$\therefore \vec{\alpha} = \dot{\vec{\omega}} = -1.8\hat{j} \text{ rad/s}$$

now solve for $\vec{a}_A$.

$$\vec{a}_A = \underbrace{\cancelto{\phi}{\vec{a}_0}}_{\text{Translation of ref. frame}} + \underbrace{\dot{\vec{\omega}} \times \vec{r}}_{\text{Tangential}} + \underbrace{\vec{\omega} \times (\vec{\omega} \times \vec{r})}_{\text{normal}} + 2\vec{\omega} \times \dot{\vec{r}}_{A/0} + \ddot{\vec{r}}_{A/0}$$

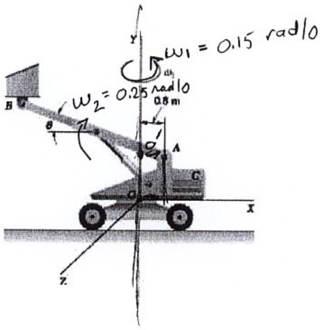
$$= \phi + (0 \;\; -1.8 \;\; 0) \times (120 \;\; 0 \;\; 180) + (1.2 \;\; 0 \;\; 1.5) \times [(1.2 \;\; 0 \;\; 1.5) \times (120 \;\; 0 \;\; 180)] + 2(1.2 \;\; 0 \;\; 1.5) \times (60 \;\; 0 \;\; 0$$

$$\vec{a}_A = \boxed{-270\hat{i} + 180\hat{j} + 172.8\hat{k} \text{ mm/s}}$$

PROBLEM 15.254

The arm *AB* of length 5 m is used to provide an elevated platform for construction workers. In the position shown, arm *AB* is being raised at the constant rate $d\theta / dt = 0.25$ rad/s; simultaneously, the unit is being rotated counterclockwise about the *Y* axis at the constant rate $\omega_1 = 0.15$ rad/s. Knowing that $\theta = 20°$, determine the velocity and acceleration of point *B*.

Reference frame rotates with $\vec{\omega}_1$ but not w/ $\vec{\omega}_2$ @ position A_{xyz}.



velocity:

$$\vec{v}_B = \vec{v}_A + \vec{\omega}_1 \times \vec{r}_{B/A} + \dot{\vec{r}}_{B/A}$$

$\vec{v}_A$ = translation of reference frame (i.e translation of ref. frame from pt O to pt A)

$$= \vec{\omega}_1 \times \vec{r}_{O/A}$$

$\dot{\vec{r}}_{B/A}$ = vel of B relative to the reference frame (since B has a rotation of $\vec{\omega}_2$ independant of A_{xyz} — rotates with $\vec{\omega}_1$)

$$= \vec{\omega}_2 \times \vec{r}_{B/A}$$

$$\therefore \vec{v}_B = \vec{\omega}_1 \times \vec{r}_{O/A} + \vec{\omega}_1 \times \vec{r}_{B/A} + \vec{\omega}_2 \times \vec{r}_{B/A}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0.15 & 0 \\ 0.8 & 0 & 0 \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0.15 & 0 \\ -4.7 & 1.71 & 0 \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & -0.25 \\ -4.7 & 1.71 & 0 \end{vmatrix} = \boxed{-0.428\hat{i} + 1.175\hat{j} + 0.585\hat{k} \text{ m/s}}$$

15.254 cont'd)

acceleration:

$$\vec{a}_B = \vec{a}_A + \cancelto{0}{\dot{\vec{\omega}}_1} \times \vec{r}_{B/A} + \vec{\omega}_1 \times (\vec{\omega}_1 \times \vec{r}_{B/A}) + 2\vec{\omega}_1 \times \dot{\vec{r}}_{B/A} + (\ddot{\vec{r}}_{B/A})$$

$$\vec{a}_A = \vec{\omega}_1 \times (\vec{\omega}_1 \times r_{A/O})$$

$$\dot{\vec{r}}_{B/A} = \vec{\omega}_2 \times \vec{r}_{B/A}$$

$$\ddot{\vec{r}}_{B/A} = \vec{\omega}_2 \times (\omega_2 \times \vec{r}_{B/A}) = \vec{\omega}_2 \times \dot{\vec{r}}_{B/A}$$

$$\therefore \vec{a}_B = \vec{\omega}_1 \times (\vec{\omega}_1 \times \vec{r}_{A/O}) + \vec{\omega}_1 \times (\vec{\omega}_1 \times \vec{r}_{B/A}) + 2\vec{\omega}_1 \times (\vec{\omega}_2 \times \vec{r}_{B/A}) + \vec{\omega}_2 \times (\vec{\omega}_2 \times \vec{r}_{B/A})$$

$$\vec{a}_B = \boxed{0.381\,\hat{\imath} + 0.1069\,\hat{\jmath} - 0.1283\,\hat{k} \ \ m/s^2}$$