PROBLEM 12.101

It was observed that during its second flyby of the earth, the Galileo spacecraft had a velocity of 14.1 km/s as it reached its minimum altitude of 303 km above the surface of the earth. Determine the eccentricity of the trajectory of the spacecraft during this portion of its flight.

SOLUTION

For earth, $R = 6.37 \times 10^6$ m

$$r_0 = 6.37 \times 10^6 + 303. \times 10^3 = 6.673 \times 10^6 \text{ m}$$

$$h = r_0 v_0 = \left(6.673 \times 10^6\right)\left(14.1 \times 10^3\right) = 94.09 \times 10^9 \text{ m}^2/\text{s}$$

$$GM = gR^2 = (9.81)\left(6.37 \times 10^6\right)^2 = 398.06 \times 10^{12} \text{ m}^3/\text{s}^2$$

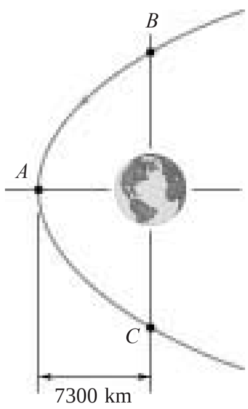
$$\frac{1}{r_0} = \frac{GM}{h^2}(1 + \varepsilon)$$

$$1 + \varepsilon = \frac{h^2}{r_0 GM} = \frac{\left(94.09 \times 10^9\right)^2}{\left(6.673 \times 10^6\right)\left(398.06 \times 10^{12}\right)} = 3.33$$

$$\varepsilon = 3.33 - 1$$

$\varepsilon = 2.33$ ◀

PROBLEM 12.112



It was observed that during its first flyby of the earth, the Galileo spacecraft had a velocity of 10.5 km/s as it reached its minimum distance of 7300 km from the center of the earth. Assuming that the trajectory of the spacecraft was parabolic, determine the time needed for the spacecraft to travel from B to C on its trajectory.

SOLUTION

For earth, $R = 6370 \text{ km} = 6.37 \times 10^6 \text{ m}, \quad g = 9.81 \text{ m/s}^2$

$$GM = gR^2 = (9.81)\left(6.37 \times 10^6\right)^2 = 3.9806 \times 10^{14} \text{ m}^3/\text{s}^2$$

At point A, $r_A = 7300 \text{ km} = 7.3 \times 10^6 \text{ m}$

$$v_A = 10.5 \text{ km/s} = 10500 \text{ m/s}$$

from which $\quad h = r_A v_A = 76.65 \times 10^9 \text{ m}^2/\text{s}$

For trajectory BAC, $\quad \dfrac{1}{r} = \dfrac{GM}{h^2}(1 + \varepsilon \cos\theta)$ with $\varepsilon = 1$

At point A, $\theta = 0$ while at B and C, $\theta = \pm 90°$

$$\frac{1}{r_B} = \frac{1}{r_C} = \frac{GM}{h^2} \qquad \text{or} \qquad r_B = r_C = \frac{h^2}{GM} = 2r_A$$

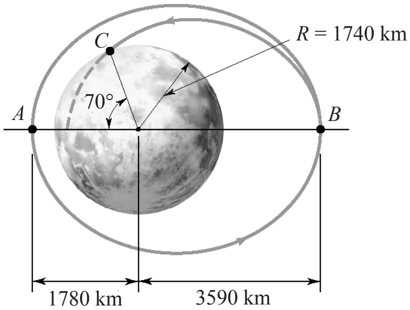
As the spacecraft travels from B to C, the area swept out is a parabolic area A.

$$A = \frac{2}{3}(r_B + r_C)r_A = \frac{8}{3}r_A^2 = \frac{8}{3}\left(7.3 \times 10^6\right)^2 = 14.21 \times 10^{13} \text{ m}^2$$

$$\frac{dA}{dt} = \frac{1}{2}h \qquad \text{or} \qquad A = \frac{1}{2}\int h\,dt = \frac{1}{2}ht$$

$$t_{BC} = \frac{2A}{h} = \frac{(2)\left(14.21 \times 10^{13}\right)}{76.65 \times 10^9} = 370\,7.76 \text{ s}$$

$\tau_{BC} = 1.03 \text{ h}$ ◀



PROBLEM 12.115

Prior to the Apollo missions to the moon, several Lunar Orbiter spacecraft were used to photograph the lunar surface to obtain information regarding possible landing sites. At the conclusion of each mission, the trajectory of the spacecraft was adjusted so that the spacecraft would crash on the moon to further study the characteristics of the lunar surface. Shown below is the elliptic orbit of Lunar Orbiter 2. Knowing that the mass of the moon is 0.01230 times the mass of the earth, determine the amount by which the speed of the orbiter should be reduced at point B so that it impacts the lunar surface at point C. (*Hint*. Point B is the apogee of the elliptic impact trajectory.)

SOLUTION

For earth, $R = 6370 \text{ km} = 6.37 \times 10^6 \text{ m}$

$$GM = gR^2 = (9.81)\left(6.37\times 10^6\right)^2 = 3.9806\times 10^{14} \text{ m}^3/\text{s}^2$$

For the moon, $GM = (0.01230)\left(3.9806\times 10^{14}\right) = 4.896\times 10^{12} \text{ m}^3/\text{s}^2$

For elliptic orbit AB, $r_A = 1780 \text{ km} = 1.78\times 10^6 \text{ m}$, $r_B = 3590 \text{ km} = 3.59\times 10^6 \text{ m}$

Using Eq. (12.39), $\dfrac{1}{r_A} = \dfrac{GM}{h^2} + C\cos\theta_A$ and $\dfrac{1}{r_B} = \dfrac{GM}{h^2} + C\cos\theta_B.$

But $\theta_B = \theta_A + 180°$, so that $\cos\theta_B = -\cos\theta_A$

Adding, $\dfrac{1}{r_A} + \dfrac{1}{r_B} = \dfrac{r_A + r_B}{r_A r_B} = \dfrac{2GM}{h^2}$

$$h = \sqrt{\frac{2GMr_Ar_B}{r_A + r_B}} = \sqrt{\frac{(2)\left(4.896\times 10^{12}\right)\left(1.78\times 10^6\right)\left(3.59\times 10^6\right)}{5.37\times 10^6}} = 3.414\times 10^9 \text{ m}^2/\text{s}$$

$$(v_B)_1 = \frac{h_{AB}}{r_A} = \frac{3.414\times 10^9}{3.59\times 10^6} = 950.9 \text{ m/s}$$

For crash trajectory BC, $\dfrac{1}{r} = \dfrac{GM}{h^2}(1 + \varepsilon\cos\theta)$

At B, $\theta = 180°$, $r = r_B$, $\dfrac{1}{r_B} = \dfrac{GM}{h_{BC}^2}(1 - \varepsilon)$ (1)

At C, $\theta = 70°$, $r = r_C$, $\dfrac{1}{r_C} = \dfrac{GM}{h_{BC}^2}(1 + \varepsilon\cos 70°)$ (2)

PROBLEM 12.115 CONTINUED

Dividing Eq. (2) by Eq. (1),

$$\frac{r_B}{r_C} = \frac{1 + \varepsilon \cos 70^\circ}{1 - \varepsilon} \qquad \text{or} \qquad \varepsilon = \frac{r_B / r_C - 1}{\left(r_B / r_C\right) + \cos 70^\circ}$$

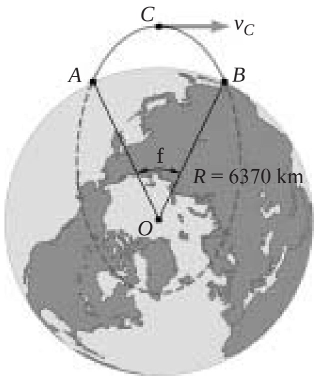
$$\varepsilon = \frac{3590/1740\ -1}{3590/1740 + \cos 70^\circ} = 0.443$$

From Eq. (1), $h_{BC} = \sqrt{GM(1-\varepsilon) r_B} = \sqrt{\left(4.896 \times 10^{12}\right)(0.557)\left(3.59 \times 10^{6}\right)} = 3.13 \times 10^{9}\ \text{m}^2/\text{s}$

$$(v_B)_2 = \frac{h_{BC}}{r_B} = \frac{3.13 \times 10^9}{3.59 \times 10^6} = 871.87\ \text{m/s}$$

$$\Delta v_B = \left(v_B\right)_2 - \left(v_B\right)_1 = -79.03\ \text{m/s}$$

$|\Delta v_B| = 79\ \text{m/s}$ ◀



PROBLEM 12.116

A long-range ballistic trajectory between points A and B on the earth's surface consists of a portion of an ellipse with the apogee at point C. Knowing that point C is 1500 km above the surface of the earth and the range $R\phi$ of the trajectory is 6000 km, determine (*a*) the velocity of the projectile at C, (*b*) the eccentricity ε of the trajectory.

SOLUTION

For earth, $R = 6370 \text{ km} = 6.37 \times 10^6 \text{ m}$

$$GM = gR^2 = (9.81)\left(6.37 \times 10^6\right)^2 = 398.06 \times 10^{12} \text{ m}^3/\text{s}^2$$

For the trajectory, $r_C = 6370 + 1500 = 7870 \text{ km} = 7.87 \times 10^6 \text{ m}$

$$r_A = r_B = R = 6.37 \times 10^6 \text{ m}, \qquad \frac{r_C}{r_A} = \frac{7870}{6370} = 1.23548$$

Range A to B: $s_{AB} = 6000 \text{ km} = 6.00 \times 10^6 \text{ m}$

$$\varphi = \frac{s_{AB}}{R} = \frac{6.00 \times 10^6}{6.37 \times 10^6} = 0.94192 \text{ rad} = 53.968°$$

For an elliptic trajectory, $\dfrac{1}{r} = \dfrac{GM}{h^2}(1 + \varepsilon\cos\theta)$

At A, $\theta = 180° - \dfrac{\varphi}{2} = 153.016°$, $\dfrac{1}{r_A} = \dfrac{GM}{h^2}(1 + \varepsilon\cos 153.016°)$ (1)

At C, $\theta = 180°$, $\dfrac{1}{r_C} = \dfrac{GM}{h^2}(1 - \varepsilon)$ (2)

Dividing Eq. (1) by Eq. (2),

$$\frac{r_C}{r_A} = \frac{1 + \varepsilon\cos 153.016°}{1 - \varepsilon} = 1.23548$$

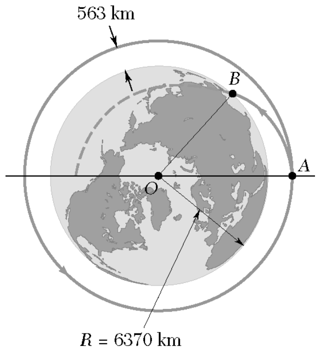
$$\varepsilon = \frac{1.23548 - 1}{1.23548 + \cos 153.016°} = 0.68384$$

From Eq. (2), $h = \sqrt{GM(1 - \varepsilon)r_C}$

$$h = \sqrt{\left(398.06 \times 10^{12}\right)(0.31616)\left(7.87 \times 10^6\right)} = 31.471 \times 10^9 \text{ m}^2/\text{s}$$

(*a*) Velocity at C. $v_C = \dfrac{h}{r_C} = \dfrac{31.471 \times 10^9}{7.87 \times 10^6} = 4.00 \times 10^3 \text{ m/s}$ $v_C = 4 \text{ km/s}$ ◀

(*b*) Eccentricity of trajectory. $\varepsilon = 0.684$ ◀



PROBLEM 12.117

A space shuttle is describing a circular orbit at an altitude of 563 km above the surface of the earth. As it passes through point A, it fires its engine for a short interval of time to reduce its speed by 152 m/s and begin its descent toward the earth. Determine the angle AOB so that the altitude of the shuttle at point B is 121 km. (*Hint*. Point A is the apogee of the elliptic descent orbit.)

SOLUTION

$$GM = gR^2 = (9.81)\left(6.37\times10^6\right)^2 = 398.06\times10^{12}\ \text{m}^3/\text{s}^2$$

$$r_A = 6370 + 563 = 6933\ \text{km} = 6.933\times10^6\ \text{m}$$

$$r_B = 6370 + 121 = 6491\ \text{km} = 6.491\times10^6\ \text{m}$$

For the circular orbit through point A,

$$v_{\text{circ}} = \sqrt{\frac{GM}{r_A}} = \sqrt{\frac{398.06\times10^{12}}{6.933\times10^6}} = 7.5773\times10^3\ \text{m/s}$$

For the descent trajectory,

$$v_A = v_{\text{circ}} + \Delta v = 7.5773\times10^3 - 152 = 7.4253\times10^3\ \text{m/s}$$

$$h = r_A v_A = \left(6.933\times10^6\right)\left(7.4253\times10^3\right) = 51.4795\times10^9\ \text{m}^2/\text{s}$$

$$\frac{1}{r} = \frac{GM}{h^2}(1+\varepsilon\cos\theta)$$

At point A, $\theta = 180°$, $r = r_A$

$$\frac{1}{r_A} = \frac{GM}{h^2}(1-\varepsilon)$$

$$1-\varepsilon = \frac{h^2}{GM\ r_A} = \frac{\left(51.4795\times10^9\right)^2}{\left(398.06\times10^{12}\right)\left(6.933\times10^6\right)} = 0.96028$$

$$\varepsilon = 0.03972$$

$$\frac{1}{r_B} = \frac{GM}{h^2}(1+\varepsilon\cos\theta_B)$$

$$1+\varepsilon\cos\theta_B = \frac{h^2}{GM\ r_B} = \frac{\left(51.4795\times10^9\right)^2}{\left(398.06\times10^{12}\right)\left(6.491\times10^6\right)} = 1.02567$$

$$\cos\theta_B = \frac{1.02567-1}{\varepsilon} = 0.6463$$

$\theta_B = 49.7°$ $\qquad$ $\measuredangle AOB = 180° - \theta_B = 130.3°$ $\qquad$ $\measuredangle AOB = 130.3°$ ◀