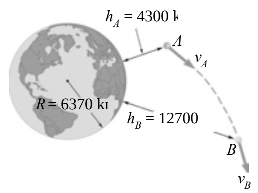
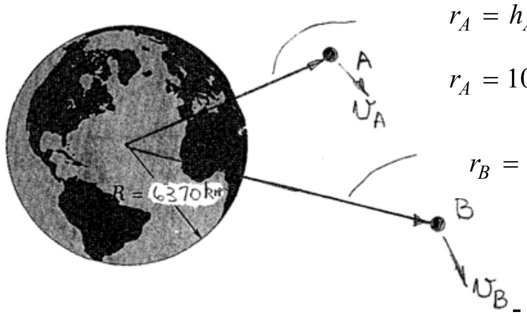
PROBLEM 13.83



Knowing that the velocity of an experimental space probe fired from the earth has a magnitude $v_A = 32.5$ Mm/h at point A, determine the velocity of the probe as it passes through point B.

SOLUTION



$$r_A = h_A + R = 4.3Mm + 6.37Mm$$

$$r_A = 10.67Mm$$

$$r_B = h_B + R = 72.7Mm + 6.37Mm$$

$$r_B = 19.07Mm$$

At A,

$$v_A = 32.5 \text{ M}m/\text{h} = 9028 \text{ m/s}$$

$$T_A = \frac{1}{2}m(9028 \text{ m/s})^2 = 40.752 \times 10^6 \text{ m}$$

$$V_A = -\frac{GMm}{r_A} = \frac{-gR^2m}{r_A}$$

$$r_A = 10.67 \text{ M}m = 10.67 \times 10^6 \text{ m}$$

$$R = 6370 \text{ km} = 6.37 \times 10^6 \text{ m}$$

$$V_A = -\frac{(9.81 \text{ m/s}^2)(6.37 \times 10^6 \text{ m})^2}{(10.67 \times 10^6 \text{ m})}m = -37.306 \times 10^6 m$$

At B

$$T_B = \frac{1}{2}mv_B^2; \; V_B = -\frac{GMm}{r_B} = \frac{-gR^2m}{r_B}$$

$$r_B = 19.07 \text{ M}m = 19.07 \times 10^6 \text{ m}$$

$$V_B = -\frac{(9.81 \text{ m/s}^2)(6.37 \times 10^6 \text{ m})^2 m}{(19.07 \times 10^6 \text{ m})} = -20.874 \times 10^6 m$$

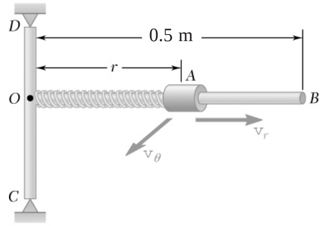
$$T_A + V_A = T_B + V_B; \; 40.752 \times 10^6 m - 37.306 \times 10^6 m = \frac{1}{2}mv_B^2 - 20.874 \times 10^6 m$$

$$v_B^2 = 2\left[40.752 \times 10^6 - 37.306 \times 10^6 + 20.874 \times 10^6\right]$$

$$v_B^2 = 48.64 \times 10^6 \text{ m}^2/\text{s}^2$$

$$v_B = 6.9742 \times 10^3 \text{ m/s} = 25.107 \text{ Mm/h}$$

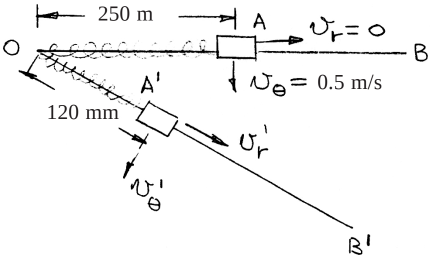
$v_B = 25.1$ Mm/h ◀



PROBLEM 13.95

A 2.5 kg collar *A* is attached to a spring of constant 750 N/m and undeformed length 150 mm. The spring is attached to point *O* of the frame *DCOB*. The system is set in motion with $r = 250$ mm, $v_\theta = 0.5$ m/s, and $v_r = 0$. Neglecting the mass of the rod and the effect of friction, determine the radial and transverse components of the velocity of the collar when $r = 120$ mm.

SOLUTION



Conservation of angular momentum about O

$$mv_\theta (0.25) = mv'_\theta (0.12)$$

$$v'_\theta = \frac{0.25\, v_\theta}{0.12} = 2.08\ (0.5) = 1.04 \text{ m/s}$$

$$v'_\theta = 1.04 \text{ m/s} \blacktriangleleft$$

Conservation of energy

$$T + V = T' + V'$$

$$T = \frac{1}{2} m\,(v_r^2 + v_\theta^2) = \frac{2.5}{2}\,(0 + 0.5^2) = 0.3125$$

$$V = \frac{1}{2} k\,(r - r_B)^2 = \frac{1}{2}\,(750)\,(0.25 - 0.15)^2 = 3.75$$

$$v'_\theta = 1.04 \text{ m/s}$$

$$T' = \frac{1}{2} m\,(v_\theta'^2 + v_r'^2) = \frac{2.5}{2}\,(v_r'^2 \;+ 1.04) = (1.25 v_r'^2 \;+ 1.352)$$

$$V' = \frac{1}{2} k\,(r' - r_B)^2 = \frac{1}{2}\,(750 \text{ N/m})\,(0.12 - 0.15)^2 = 0.3375 \text{ N.m}$$

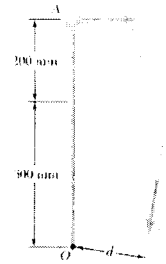
$$T + V = T' + V' : 0.3125 + 3.75 = 1.25 v_r'^2 + 1.352 + 0.3375$$

$$0.3125 + 3.75 = 1.25 v_r'^2 + 1.352 + 0.3375$$

$$1.25 v_r'^2 = 2.373$$

$$v_r'^2 = 1.90$$

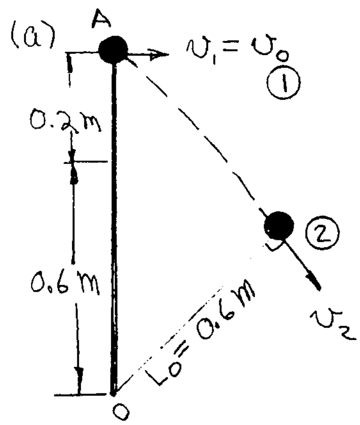
$$v'_r = 1.38 \text{ m/s} \blacktriangleleft$$



PROBLEM 13.97

A 0.7-kg ball that can slide on a *horizontal* frictionless surface is attached to a fixed point O by means of an elastic cord of constant $k = 150$ N/m and undeformed length 600 mm. The ball is placed at point A, 800 mm from O, and given an initial velocity $\mathbf{v}_0$ perpendicular to OA. Determine (a) the smallest allowable value of the initial speed v_0 if the cord is not to become slack, (b) the closest distance d that the ball will come to point O if it is given half the initial speed found in part a.

SOLUTION



The cord will not go slack if v_2 is perpendicular to the undeformed cord length, L_0, at ②

Conservation of angular momentum

$$0.8v_1 = 0.6v_2 \qquad v_2 = \frac{0.8}{0.6}v_1 = 1.333v_0$$

Conservation of energy

Point ① $\qquad v_1 = v_0 \qquad T_1 = \frac{1}{2}mv_0^2 = 0.35v_0^2$

$$V_1 = \frac{1}{2}k(L - L_0)^2 = \frac{1}{2}(150\text{ N/m})(0.8\text{ m} - 0.6\text{ m})^2$$

$$V_1 = 3\text{ J}$$

Point ② $\qquad T_2 = \frac{1}{2}mv_2^2 = 0.35v_2^2$

$$\Delta L = 0 \qquad V = 0 \qquad T_1 + V_1 = T_2 + V_2:\ 0.35v_0^2 + 3 = 0.35v_2^2 + 0$$

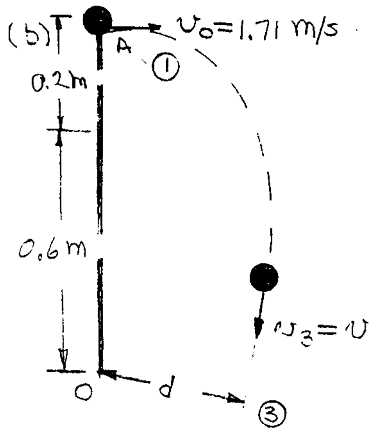
From conservation of angular momentum $\qquad v_2 = \sout{1.3158v_B}$ 1.333 v_0

$$0.35v_0^2\left[(\sout{1.3158}\ 1.333)^2 - 1\right] = 3$$

$$v_0^2 = \frac{(3\text{ J})}{(0.35\text{ kg})(\sout{0.7313}\ 0.777)} = \sout{11.72}\ 11.02\text{ m}^2/\text{s}^2$$

$v_0 = \sout{3.42}$ 3.32 m/s ◀

PROBLEM 13.97 CONTINUED



The ball travels in a straight line after the cord goes slack.

Conservation of angular momentum

$$(0.8)(\cancel{1.71}\ 1.66) = dv$$

$$d = \frac{\cancel{1.368}\ 1.328}{v}$$

Conservation of energy

$$v_1 = \cancel{1.71}\ 1.66 \text{ m/s}$$

Point ①

$$T_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(0.7 \text{ kg})(\cancel{1.71}\ 1.66 \text{ m/s})^2 = \cancel{1.0234}\ 0.9645 \text{ J}$$

$$V_1 = \frac{1}{2}k(L - L_0)^2 = \frac{1}{2}(150 \text{ N/m})(0.8 \text{ m} - 0.6 \text{ m})^2 = 3 \text{ J}$$

Point ③

$$T_3 = \frac{1}{2}mv_3^2 = 0.35v^2$$

$$V_3 = 0$$

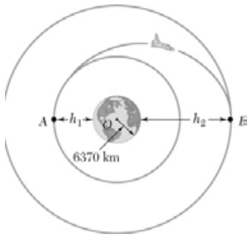
$$T_1 + V_1 = T_3 + V_3: \ \cancel{1.0234}\ 0.9645 + 3 = 0.35v^2 + 0$$

$$v = \cancel{3.39}\ 3.366 \text{ m/s}$$

From conservation of momentum

$$d = \frac{\cancel{1.368}\ 1.328}{v} = \frac{\cancel{1.368}\ 1.328}{\cancel{3.39}\ 3.366} = \cancel{404}\ 394.6 \text{ mm}$$

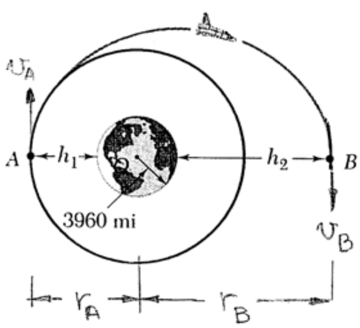
$$d = \cancel{404}\ 394.6 \text{ mm} \blacktriangleleft$$



PROBLEM 13.105

The optimal way of transferring a space vehicle from an inner circular orbit to an outer coplanar orbit is to fire its engines as it passes through *A* to increase its speed and place it in an elliptic transfer orbit. Another increase in speed as it passes through *B* will place it in the desired circular orbit. For a vehicle in a circular orbit about the earth at an altitude $h_1 = 320$ km, which is to be transferred to a circular orbit at an altitude $h_2 = 800$ km, determine (*a*) the required increase in speed at *A* and *B*, (*b*) the total energy per unit mass required to execute the transfer.

SOLUTION



Elliptical orbit between A and B

Conservation of angular momentum

$$mr_A v_A = mr_B v_B$$

$$v_A = \frac{r_B}{r_A} v_B = \frac{7.170}{6.690} v_B$$

$$r_A = 6370 \text{ km} + 320 \text{ km} = 6690 \text{ km}, \qquad r_A = 6.690 \times 10^6 \text{ m}$$

$$v_A = 1.0718 v_B \tag{1}$$

$$r_B = 6370 \text{ km} + 800 \text{ km} = 7170 \text{ km}, \qquad r_B = 7.170 \times 10^6 \text{ m}$$

$$R = (6370 \text{ km}) = 6.37 \times 10^6 \text{ m}$$

Conservation of energy

$$GM = gR^2 = (9.81 \text{ m/s}^2)(6.37 \times 10^6 \text{ m})^2 = 398.060 \times 10^{12} \text{ m}^3/\text{s}^2$$

Point A

$$T_A = \frac{1}{2} m v_A^2 \qquad V_A = -\frac{GMm}{r_A} = -\frac{(398.060 \times 10^{12})m}{(6.690 \times 10^6)}$$

$$V_A = 59.501 \times 10^6 m$$

Point B

$$T_B = \frac{1}{2} m v_B^2 \qquad V_B = -\frac{GMm}{r_B} = -\frac{(398.060 \times 10^{12})m}{(7.170 \times 10^6)}$$

$$V_B = 55.5 \times 10^6 m$$

$$T_A + V_A = T_B + V_B$$

PROBLEM 13.105 CONTINUED

$$\frac{1}{2}mv_A^2 - 59.501 \times 10^6 m = \frac{1}{2}mv_B^2 - 55.5 \times 10^6 m$$

$$v_A^2 - v_B^2 = 8.002 \times 10^6$$

From (1) $\quad v_A = 1.0718 v_B \qquad v_B^2\left[(1.0718)^2 - 1\right] = 8.002 \times 10^6$

$$v_B^2 = 53.79 \times 10^6 \text{ m}^2/\text{s}^2, \qquad v_B = 7334 \text{ m/s}$$

$$v_A = (1.0718)(7334 \text{ m/s}) = 7861 \text{ m/s}$$

Circular orbit at A and B

(Equation 12.44)

$$(v_A)_C = \sqrt{\frac{GM}{r_A}} = \sqrt{\frac{398.060 \times 10^{12}}{6.690 \times 10^6}} = 7714 \text{ m/s}$$

$$(v_B)_C = \sqrt{\frac{GM}{r_B}} = \sqrt{\frac{398.060 \times 10^{12}}{7.170 \times 10^6}} = 7451 \text{ m/s}$$

(*a*) Increases in speed at A and B

$$\Delta v_A = v_A - (v_A)_C = 7861 - 7714 = 147 \text{ m/s} \blacktriangleleft$$

$$\Delta v_B = (v_B)_C - v_B = 7451 - 7334 = 117 \text{ m/s} \blacktriangleleft$$

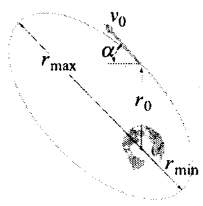
(*b*) Total energy per unit mass

$$E/m = \frac{1}{2}\left[(v_A)^2 - (v_A)_C^2 + (v_B)_C^2 - (v_B)^2\right]$$

$$E/m = \frac{1}{2}\left[(7861)^2 - (7714)^2 + (7451)^2 - (7334)^2\right]$$

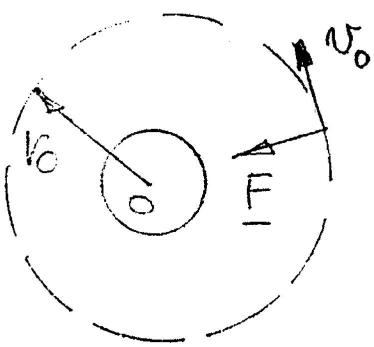
$$E/m = 2.01 \times 10^6 \text{ J/kg} \blacktriangleleft$$

PROBLEM 13.113



A satellite is projected into space with a velocity $\mathbf{v}_0$ at a distance r_0 from the center of the earth by the last stage of its launching rocket. The velocity $\mathbf{v}_0$ was designed to send the satellite into a circular orbit of radius r_0. However, owing to a malfunction of control, the satellite is not projected horizontally but at an angle α with the horizontal and, as a result, is propelled into an elliptic orbit. Determine the maximum and minimum values of the distance from the center of the earth to the satellite.

SOLUTION



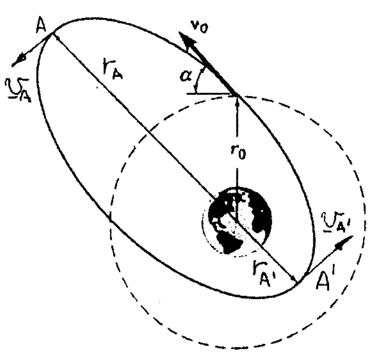
For circular orbit of radius r_0

$$F = ma_n \qquad \frac{GMm}{r_0^2} = m\frac{v_0^2}{r_0}$$

$$v_0^2 = \frac{GM}{r_0}$$

But v_0 forms an angle α with the intended circular path

For elliptic orbit



Conservation of angular momentum

$$r_0 m v_0 \cos\alpha = r_A m v_A$$

$$v_A = \left(\frac{r_0}{r_A}\cos\alpha\right)v_0 \qquad (1)$$

Conservation of energy

$$\frac{1}{2}mv_0^2 - \frac{GMm}{r_0} = \frac{1}{2}mv_A^2 - \frac{GMm}{r_A}$$

$$v_0^2 - v_A^2 = \frac{2GM}{r_0}\left(1 - \frac{r_0}{r_A}\right)$$

Substitute for v_A from (1)

$$v_0^2\left[1 - \left(\frac{r_0}{r_A}\right)^2\cos^2\alpha\right] = \frac{2GM}{r_0}\left(1 - \frac{r_0}{r_A}\right)$$

But $v_0^2 = \dfrac{GM}{r_0}$ thus $\quad 1 - \left(\dfrac{r_0}{r_A}\right)^2\cos^2\alpha = 2\left(1 - \dfrac{r_0}{r_A}\right)$

$$\cos^2\alpha\left(\frac{r_0}{r_A}\right)^2 - 2\left(\frac{r_0}{r_A}\right) + 1 = 0$$

PROBLEM 13.113 CONTINUED

Solving for $\frac{r_0}{r_A}$

$$\frac{r_0}{r_A} = \frac{+2 \pm \sqrt{4 - 4\cos^2\alpha}}{2\cos^2\alpha} = \frac{1 \pm \sin\alpha}{1 - \sin^2\alpha}$$

$$r_A = \frac{(1 + \sin\alpha)(1 - \sin\alpha)}{1 \pm \sin\alpha} r_0 = (1 \mp \sin\alpha) r_0$$

also valid for point A'

Thus

$$r_{\max} = (1 + \sin\alpha) r_0 \qquad r_{\min} = (1 - \sin\alpha) r_0 \blacktriangleleft$$