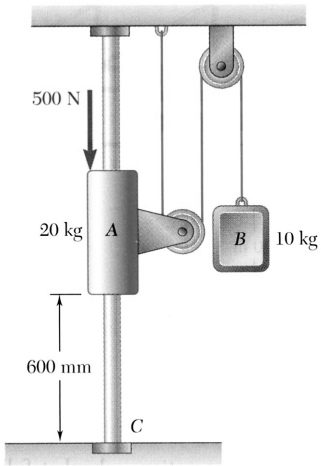
PROBLEM 13.19



The system shown, consisting of a 20-kg collar *A* and a 10-kg counterweight *B*, is at rest when a constant 500-N force is applied to collar *A*. (a) Determine the velocity of *A* just before it hits the support at *C*. (b) Solve part a assuming that the counterweight *B* is replaced by a 98.1-N downward force. Ignore friction and the mass of the pulleys.

SOLUTION

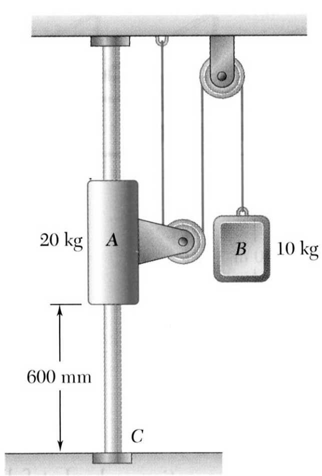
Kinematics

Displacement of *A*, $x_A = 0.6$ m

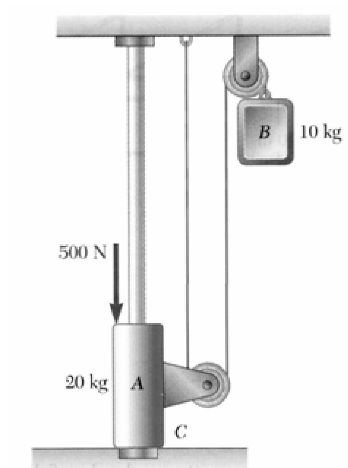
Displacement of *B*, $x_B = 2 \times 0.6 = 1.2$ m

$$x_B = 2x_A$$
$$v_B = 2v_A$$

(a)



Position 1



Position 2

Potential energy, $V_2 - V_1 = m_A g(x_{A2} - x_{A1}) + m_B g(x_{B2} - x_{B1})$

$$= 20(9.81)(-0.6) + 10(9.81)(1.2)$$
$$= 0$$

Kinetic energy, $T_2 - T_1 = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2$

$$= \frac{1}{2}(20)v_A^2 + \frac{1}{2}(10)(2v_A)^2$$

$$= 30v_A^2$$

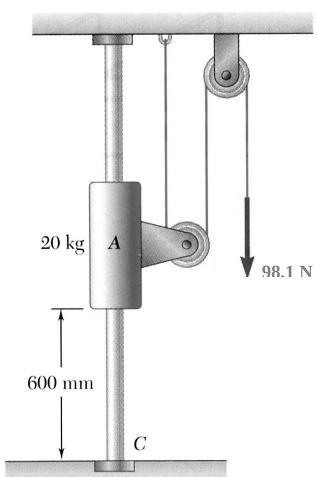
F_n is the force that does the work on the system which cause the change of position. The work done, $\int_1^2 F_n dx$ is positive if it is in the same direction as the motion.

Energy equation, $\int_1^2 F_n dx = (V_2 + T_2) - (V_1 + T_1)$

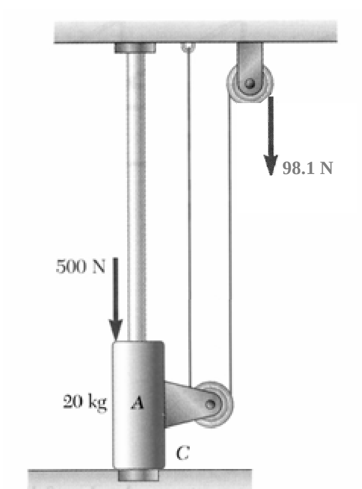
$$+500(0.6) = 0 + 30v_A^2$$

$$v_A = 3.162 \text{ m/s}$$

(b)



Position 1



Position 2

Potential energy, $V_2 - V_1 = m_A g(x_{A2} - x_{A1})$

$$= 20(9.81)(-0.6)$$

$$= -117.72 \text{ Joule}$$

Kinetic energy, $T_2 - T_1 = \frac{1}{2} m_A v_A^2$

$$= \frac{1}{2}(20)v_A^2$$

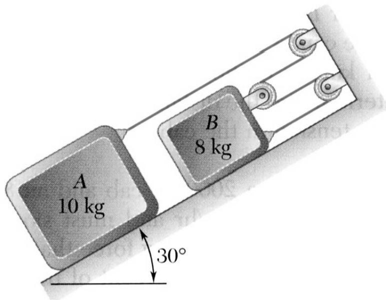
$$= 10v_A^2$$

Energy equation, $\int_1^2 F_n dx = (V_2 + T_2) - (V_1 + T_1)$

$$+500(0.6) + (-98.1)(1.2) = -117.72 + 10v_A^2$$

$$v_A = 5.477 \text{ m/s}$$

PROBLEM 13.21



The two blocks shown are released from rest. Neglecting the masses of the pulleys and the effect of friction in the pulleys and between the blocks and the incline, determine (a) the velocity of block *A* after it has moved 0.5 m, (b) tension in the cable.

SOLUTION

Kinematics

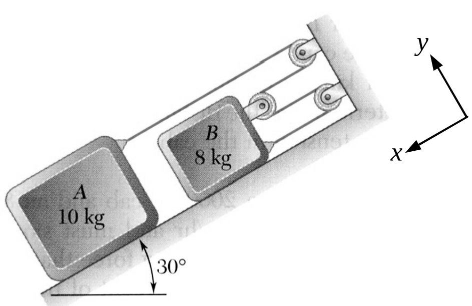
Displacement of *A*, $x_A = 0.5$ m

Displacement of *B*, $x_B = \frac{1}{3} \times 0.5 = \frac{0.5}{3}$ m

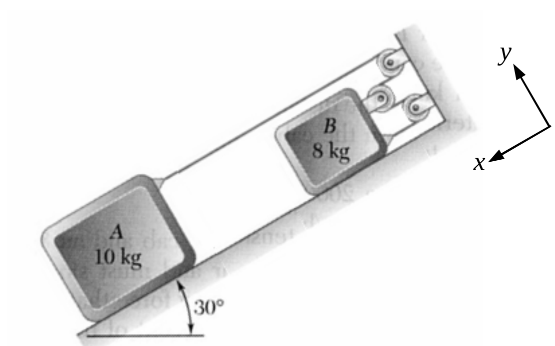
$$x_B = \frac{x_A}{3}$$

$$v_B = \frac{v_A}{3}$$

(a)



Position 1



Position 2

Potential energy, $V_2 - V_1 = m_A g x_A + m_B g x_B$

$$= 10(9.81)(-0.5)\sin 30^\circ + 8(9.81)\left(\frac{0.5}{3}\right)\sin 30^\circ$$

$$= -17.985 \text{ Joule}$$

Kinetic energy, $T_2 - T_1 = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2$

$$= \frac{1}{2}(10)v_A^2 + \frac{1}{2}(8)\left(\frac{v_A}{3}\right)^2$$

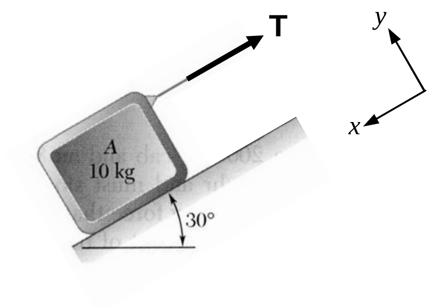
$$= \frac{49}{9} v_A^2$$

Energy equation, $\int_1^2 F_n dx = (V_2 + T_2) - (V_1 + T_1)$

$$0 = -17.985 + \frac{49}{9} v_A^2$$

$$v_A = 1.818 \text{ m/s } \angle 30^\circ$$

(b)



Energy equation of block A,

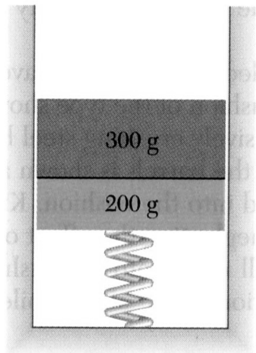
$$\int_1^2 F_n dx = (V_2 + T_2) - (V_1 + T_1)$$

$$(-T)(0.5) = m_A g \sin 30^\circ x_A + \frac{1}{2} m_A v_A^2$$

$$-0.5T = (10)(9.81)\sin 30^\circ(-0.5) + \frac{1}{2}(10)(1.818)^2$$

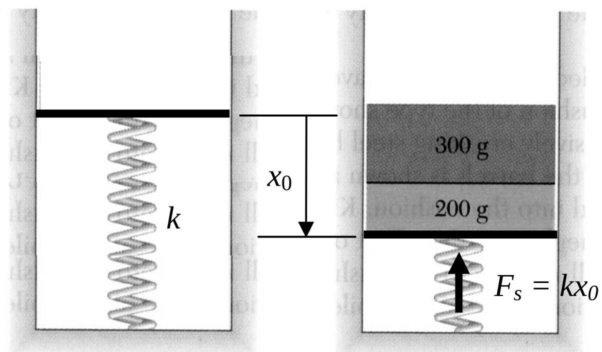
$$T = 15.999 \text{ N} \angle 30^\circ$$

PROBLEM 13.25



A 300 g block rests on top of a 200 g block supported by but not attached to a spring of constant 135 N/m. The upper block is suddenly removed. Determine (a) the maximum velocity reached by the 200 g block, (b) the maximum height reached by the 200 g block.

SOLUTION



Uncompressed position   Position 1

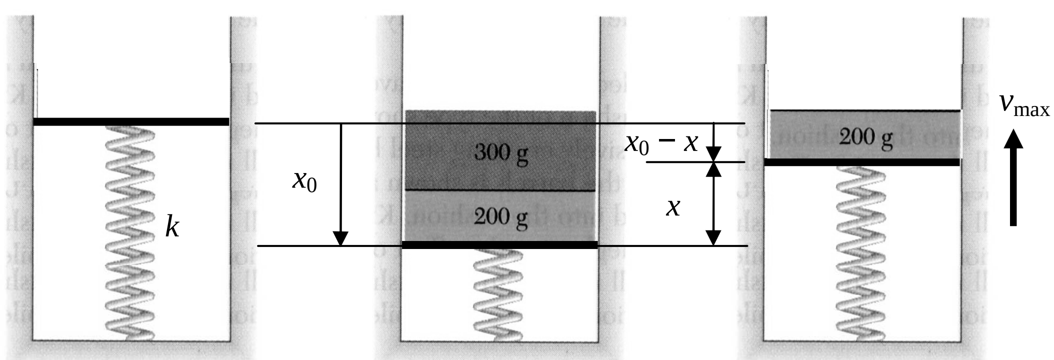
Statics

Compression height due to (300 g + 200 g), $kx_0 = (0.3 + 0.2)g$

$$x_0 = \frac{(0.5)9.81}{135}$$

$$= 0.0363 \text{ m}$$

(a) The maximum velocity will occur while the spring is still in contact with the 200 g block.



Uncompressed position   Position 1   Position 2

Potential energy, $V_1 = \frac{1}{2}kx_0^2 = \frac{1}{2}(135)(0.0363)^2 = 0.0889 \text{ Joule}$

$$V_2 = m_{200\,\text{g}}gx + \frac{1}{2}k(x_0 - x)^2 = (0.2)(9.81)x + \frac{1}{2}(135)(0.0363 - x)^2$$
$$= 67.5x^2 - 2.9385x + 0.0889$$

Therefore, $V_2 - V_1 = 67.5x^2 - 2.9385x + 0.0889 - 0.0889$
$$= 67.5x^2 - 2.9385x$$

Kinetic energy, $T_1 = 0$

$$T_2 = \frac{1}{2}m_{200\,\text{g}}v^2 = 0.1v^2$$

Therefore, $T_2 - T_1 = 0.1v^2$

Energy equation, $\int_1^2 F_n dx = (V_2 + T_2) - (V_1 + T_1)$

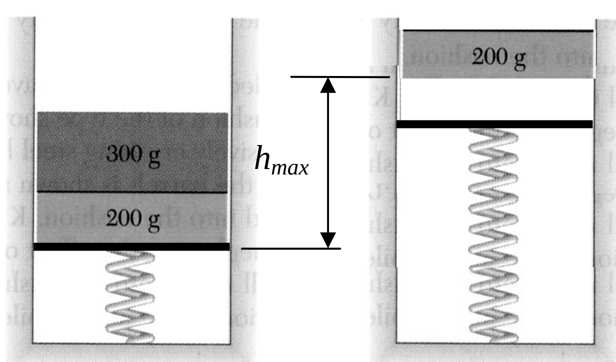
$$0 = 67.5x^2 - 2.9385x + 0.1v^2$$
$$v^2 = -675x^2 + 29.385x$$

v is maximum when $\frac{dv}{dx} = 0$

$$\frac{dv^2}{dx} = -1350x + 29.385$$
$$0 = -1350x + 29.385$$
$$x = 0.0218 \text{ m}$$

Therefore, $v_{\max}^2 = -675(0.0218)^2 + 29.385(0.0218)$
$$v_{\max} = 0.5655 \text{ m/s}$$

(b)



Position 1 Position 2

Potential energy, $V_1 = \frac{1}{2}kx_0^2 = \frac{1}{2}(135)(0.0363)^2 = 0.0889 \text{ Joule}$

$V_2 = m_{200\text{ g}}\, g x_{\max} = (0.2)(9.81)x_{\max} = 1.962x_{\max}$

Therefore, $V_2 - V_1 = 1.962x_{\max} - 0.0889$

Kinetic energy, $T_1 = 0$

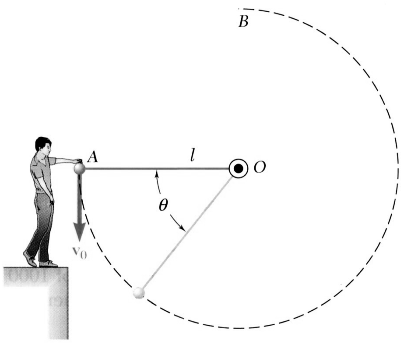
$T_2 = 0$

Energy equation, $\int_1^2 F_n dx = (V_2 + T_2) - (V_1 + T_1)$

$0 = 1.962x_{\max} - 0.0889$

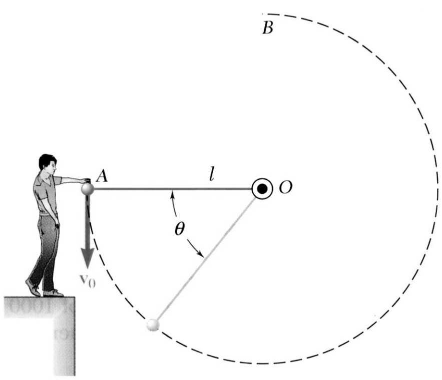
$x_{\max} = 0.0453 \text{ m}$

PROBLEM 13.39

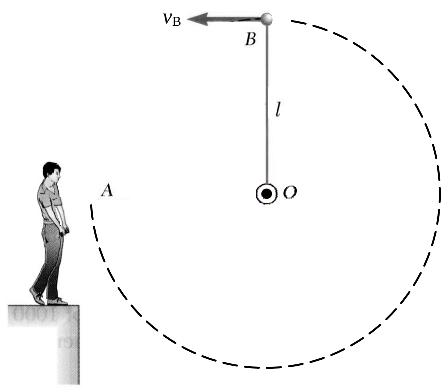


The sphere at A is given a downward velocity $\mathbf{v}_0$ and swings in a vertical circle of radius l and centre O. Determine the smallest velocity $\mathbf{v}_0$ for which the sphere will reach point B as it swings about point O (a) if AO is a rope, (b) if AO is a slender rod of negligible mass.

SOLUTION



Position 1



Position 2

Potential energy, $V_2 - V_1 = mgl$

Kinetic energy, $T_2 - T_1 = \frac{1}{2}mv_B^2 - \frac{1}{2}mv_0^2$

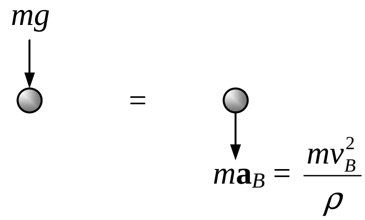
Energy equation, $\int_1^2 F_n dx = (V_2 + T_2) - (V_1 + T_1)$

$$0 = mgl + \frac{1}{2}mv_B^2 - \frac{1}{2}mv_0^2$$

$$v_0^2 = 2gl + v_B^2$$

(a) For minimum v_B, tension in the cord is zero.

Free body diagram of sphere at B:



$$F = ma$$

$$mg = \frac{mv_B^2}{\rho}$$

$$v_B^2 = gl$$

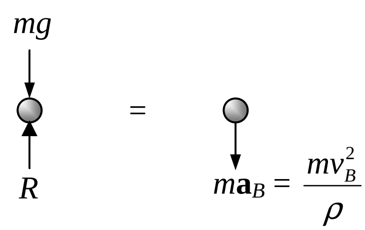
Substituting into energy equation,

$$v_0^2 = 2gl + gl$$

$$v_0 = \sqrt{3gl}$$

(b) Force in the rod can support the weight of the sphere, $R + mg = 0$.

Free body diagram of sphere at B:



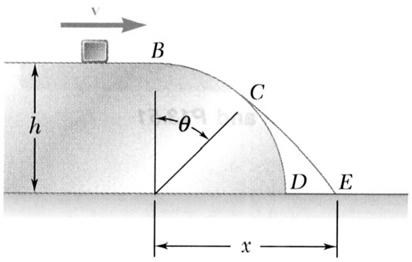
$$0 = \frac{mv_B^2}{\rho}$$

$$v_B = 0$$

Therefore, $v_0^2 = 2gl + 0$

$$v_0 = \sqrt{2gl}$$

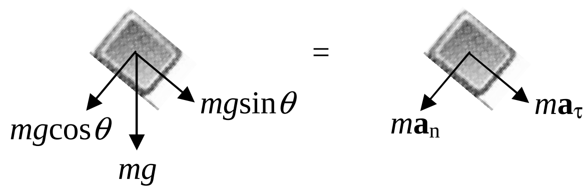
PROBLEM 13.45



A small block slides at a speed v = 3 m/s on a horizontal surface at a height h = 1m above the ground. Determine (a) the angle θ at which it will leave the cylindrical surface BCD, (b) the distance x at which it will hit the ground. Neglect friction and air resistance.

SOLUTION

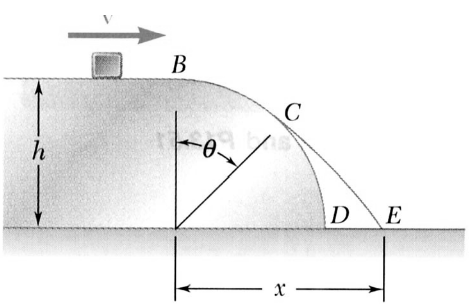
Free-body diagram at C



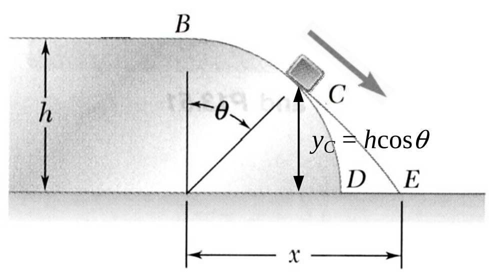
↙ - direction:

$$F = ma$$
$$mg\cos\theta = m\mathbf{a}_n$$
$$g\cos\theta = \frac{v_C^2}{\rho}$$
$$v_C^2 = gy_C \qquad \text{where } h\cos\theta = y_C$$

(a)



Position 1



Position 2

Potential energy, $V_2 - V_1 = mg(y_C - h)$
$$= 9.81m(y_C - 1)$$

Kinetic energy, $T_2 - T_1 = \frac{1}{2}mv_C^2 - \frac{1}{2}mv^2$

$$= \frac{1}{2}mgy_C - \frac{1}{2}m(3)^2 \quad \text{since } v_C^2 = gy_C$$

$$= 4.905my_C - 4.5m$$

Energy equation, $\int_1^2 F_n dx = (V_2 + T_2) - (V_1 + T_1)$

$$0 = 9.81m(y_C - 1) + 4.905my_C - 4.5m$$

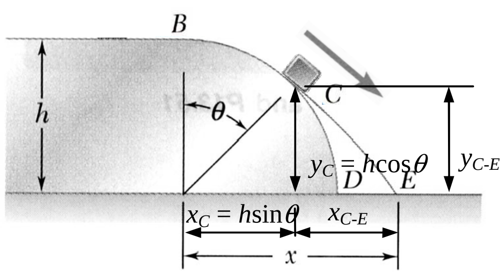
$$y_C = 0.9725 \text{ m}$$

Therefore, $y_C = h\cos\theta$

$$\cos\theta = 0.9725$$

$$\theta = 13.47^\circ$$

(b)



Since $y_C = 0.9725$ m, $v_C^2 = gy_C$

$$= 9.81(0.9725)$$

$$v_C = 3.089 \text{ m/s}$$

Projectile from C to E (y-coordinate), $y_{C-E} = (v_C)_Y t - \frac{1}{2}gt^2$

$$y_{C-E} = 3.089\left(\sin 13.47^\circ\right)t - \frac{1}{2}(9.81)t^2$$

$$y_{C-E} = 0.7195\,t - 4.905\,t^2$$

Since at C, $y_{C-E} = y_C$

$$0.7195\,t - 4.905\,t^2 = 0.9725$$

$$t = 0.3779 \text{ s}$$

Projectile from C to E (x-coordinate), $x_{C-E} = (v_C)_x t = 3.089\left(\cos 13.47^\circ\right)(0.3779) = 1.135 \text{ m}$

Thus, $x = x_C + x_{C-E}$

$$= h\sin 13.47^\circ + 1.135$$

$$= 1.368 \text{ m}$$