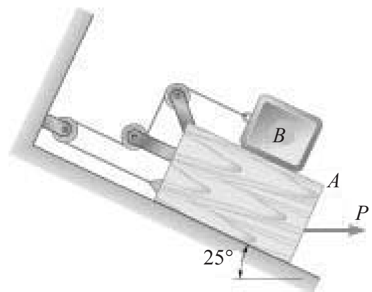
PROBLEM 12.16



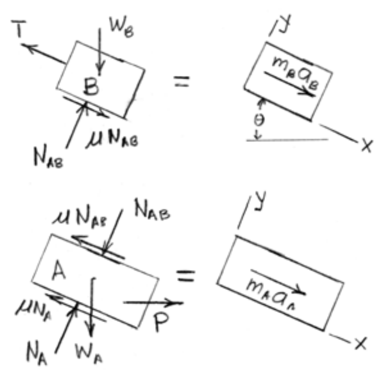
Block A has a mass of 40 kg, and block B has a mass of 8 kg. The coefficients of friction between all surfaces of contact are $\mu_s = 0.20$ $\mu_k = 0.15$. Knowing that $\mathbf{P} = 50\text{ N} \rightarrow$, determine (*a*) the acceleration of block B, (*b*) the tension in the cord.

SOLUTION

Constraint of cable: $2x_A + (x_B - x_A) = x_A + x_B = \text{constant.}$

$$a_A + a_B = 0, \qquad \text{or} \qquad a_B = -a_A$$

Assume that block A moves down and block B moves up.



Block B: $+\nearrow \Sigma F_y = 0: \quad N_{AB} - W_B\cos\theta = 0$

$$+\searrow \Sigma F_x = ma_x: \quad -T + \mu N_{AB} + W_B\sin\theta = \frac{W_B}{g}a_B$$

Eliminate N_{AB} and a_B.

$$-T + W_B(\sin\theta + \mu\cos\theta) = W_B\frac{a_B}{g} = -W_B\frac{a_A}{g}$$

Block A: $+\nearrow \Sigma F_y = 0: \quad N_A - N_{AB} - W_A\cos\theta + P\sin\theta = 0$

$$N_A = N_{AB} + W_A\cos\theta - P\sin\theta$$

$$= (W_B + W_A)\cos\theta - P\sin\theta$$

$$\Sigma F_x = m_A a_A: \quad -T + W_A\sin\theta - F_{AB} - F_A + P\cos\theta = \frac{W_A}{g}a_A$$

$$-W_B(\sin\theta + \mu\cos\theta) - W_B\frac{a_A}{g} + W_A\sin\theta - \mu W_B\cos\theta$$

$$-\mu(W_B + W_A)\cos\theta + \mu P\sin\theta + P\cos\theta = W_A\frac{a_A}{g}$$

$$(W_A - W_B)\sin\theta - \mu(W_A + 3W_B)\cos\theta + P(\mu\sin\theta + \cos\theta) = (W_A + W_B)\frac{a_A}{g}$$

Check the condition of impending motion.

$$\mu = \mu_s = 0.20, \quad a_A = a_B = 0, \quad \theta = 25^\circ$$

$$(W_A - W_B)\sin\theta - \mu_s(W_A + 3W_B)\cos\theta + P_s(\mu_s\sin\theta + \cos\theta) = 0$$

PROBLEM 12.16 CONTINUED

$$P_s = \frac{\mu_s(W_A + 3W_B)\cos\theta - (W_A - W_B)\sin\theta}{\mu_s \sin\theta + \cos\theta}$$

$$= \frac{(0.20)(64)(9.81)\cos 25° - (32)(9.81)}{0.20 \sin 25° + \cos 25°} = -19.04 \text{ N} < 50 \text{ N}$$

Blocks will move with $P = 50$ N.

Calculate $\frac{a_A}{g}$ using $\mu = \mu_k = 0.15$, $\theta = 25°$, and $P = 50$ N.

$$\frac{a_A}{g} = \frac{(W_A - W_B)\sin\theta - \mu_k (W_A + 3W_B)\cos\theta + P(\mu_k \sin\theta + \cos\theta)}{W_A + W_B}$$

$$= \frac{(32)(9.81)\sin 25° - (0.15)(64)(9.81)\cos 25° + (50)(0.15 \sin 25° + \cos 25°)}{(48)(9.81)}$$

$$= 0.203449$$

$$a_A = (0.203449)(9.81) = 1.995 \text{ m/s}^2$$

(*a*) $a_B = -1.995 \text{ m/s}^2$

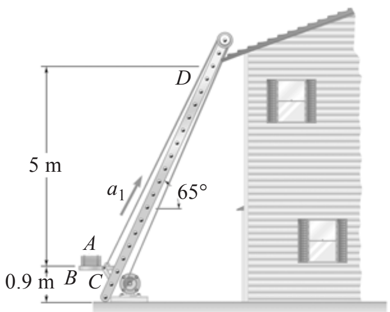
$\mathbf{a}_B = 2 \text{ m/s}^2$ ⦪ 25° ◀

(*b*) $T = W_B(\sin\theta + \mu\cos\theta) + W_B \frac{a_A}{g}$

$$= (8)(9.81)(\sin 25° + 0.15 \cos 25°) + (8)(9.81)(0.203449)$$

$T = 59.8$ N ◀

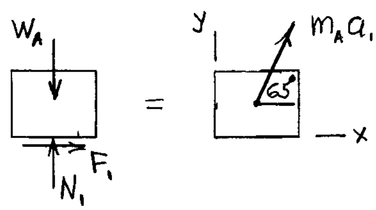
PROBLEM 12.22



To transport a series of bundles of shingles *A* to a roof, a contractor uses a motor-driven lift consisting of a horizontal platform *BC* which rides on rails attached to the sides of a ladder. The lift starts from rest and initially moves with a constant acceleration $\mathbf{a}_1$ as shown. The lift then decelerates at a constant rate $\mathbf{a}_2$ and comes to rest at *D*, near the top of the ladder. Knowing that the coefficient of static friction between the bundle of shingles and the horizontal platform is 0.30, determine the largest allowable acceleration $\mathbf{a}_1$ and the largest allowable deceleration $\mathbf{a}_2$ if the bundle is not to slide on the platform.

SOLUTION

Acceleration $\mathbf{a}_1$: Impending slip. $F_1 = \mu_s N_1 = 0.30\, N_1$



$$\Sigma F_y = m_A a_y: \quad N_1 - W_A = m_A a_1 \sin 65°$$

$$N_1 = W_A + m_A a_1 \sin 65°$$

$$= m_A\left(g + a_1 \sin 65°\right)$$

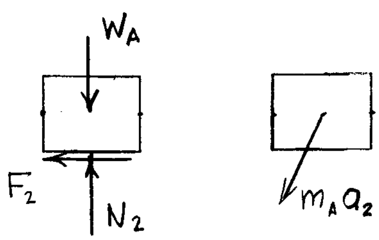
$$\xrightarrow{+} \Sigma F_x = m_A a_x: \quad F_1 = m_A a_1 \cos 65°$$

$$F_1 = \mu_s N \quad \text{or} \quad m_A a_1 \cos 65° = 0.30 m_A\left(g + a_1 \sin 65°\right)$$

$$a_1 = \frac{0.30g}{\cos 65° - 0.30 \sin 65°} = (1.990)(9.81) = 19.53 \text{ m/s}^2$$

$$\mathbf{a}_1 = 19.53 \text{ m/s}^2 \measuredangle 65° \blacktriangleleft$$

Deceleration $\mathbf{a}_2$: Impending slip. $F_2 = \mu_S N_2 = 0.30\, N_2$



$$\Sigma F_y = m a_y: \quad N_1 - W_A = -m_A a_2 \sin 65°$$

$$N_1 = W_A - m_A a_2 \sin 65°$$

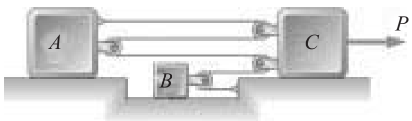
$$\xleftarrow{+} \Sigma F_x = m a_x: \quad F_2 = m_A a_2 \cos 65°$$

$$F_2 = \mu_S N_2 \qquad \text{or} \qquad m_A a_2 \cos 65° = 0.30 m_A\left(g - a_2 \cos 65°\right)$$

$$a_2 = \frac{0.30g}{\cos 65° + 0.30 \sin 65°} = (0.432)(9.81) = 4.24 \text{ m/s}^2$$

$$\mathbf{a}_2 = 4.24 \text{ m/s}^2 \nearrow 65° \blacktriangleleft$$

PROBLEM 12.29



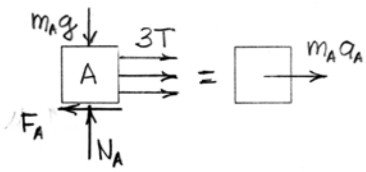
The coefficients of friction between the three blocks and the horizontal surfaces are $\mu_s = 0.25$ and $\mu_k = 0.20$. The masses of the blocks are $m_A = m_C = 10$ kg, and $m_B = 5$ kg. Knowing that the blocks are initially at rest and that C moves to the right through 0.8 m in 0.4 s, determine (*a*) the acceleration of each block, (*b*) the tension in the cable, (*c*) the force **P**. Neglect axle friction and the masses of the pulleys.

SOLUTION

Let the positive direction for position coordinates, velocities, and accelerations be to the right. Let the origin lie at the fixed anchor.

Constraint of cable: $3(x_C - x_A) + (x_C - x_B) + (-x_B) = \text{constant}$

$$4a_C - 2a_B - 3a_A = 0 \qquad (1)$$

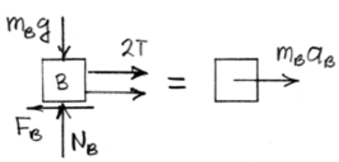


Block A: $\uparrow \Sigma F_y = 0$: $N_A - m_A g = 0$

$$N_A = m_A g, \quad F_A = \mu_k N_A = \mu_k m_A g$$

$$\xrightarrow{+} \Sigma F_x = m_A a_x: \quad 3T - F_A = m_A a_A$$

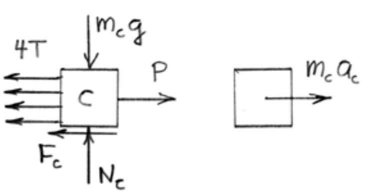
$$a_A = \frac{3T - \mu_k m_A g}{m_A} = \frac{3T}{10} - 0.20g \qquad (2)$$



Block B: $N_B = m_B g, \quad F_B = \mu_k m_B g$

$$\xrightarrow{+} \Sigma F_x = m_B a_B: \quad 2T - F_B = m_B a_B$$

$$a_B = \frac{2T - \mu_k m_B g}{m_B} = \frac{2T}{5} - 0.20g \qquad (3)$$



Block C: $N_C = m_C g, \quad F_C = \mu_k m_C g$

$$\xrightarrow{+} \Sigma F_x = m_C a_A: \quad P - 4T = m_C a_C \qquad (4)$$

Kinematics: $x_C = (x_C)_o + (v_C)_o + \frac{1}{2}a_C t^2 = 0 + \frac{1}{2}a_C t^2$

$$a_C = \frac{2\left[x_C - (x_C)_o\right]}{t^2} = \frac{(2)(0.8)}{(0.4)^2} = 10.00 \text{ m/s}^2 \qquad (5)$$

Substitute (2), (3) and (5) into (1).

$$(4)(10) - (2)\left(\frac{2T}{5} - 0.20g\right) - (3)\left(\frac{3T}{10} - 0.20g\right) = 40 - 1.7T - 2g = 0$$

PROBLEM 12.29 CONTINUED

$$T = \frac{40 - 2g}{1.7} = \frac{40 - (2)(9.81)}{1.7} = 29.294 \text{ N}$$

From (4), $P = 4T + m_C a_C = (4)(29.294) + (10)(10) = 236.80 \text{ N}$

From (2), $a_A = \frac{(3)(29.294)}{10} - (0.20)(9.81) = 6.826 \text{ m/s}^2$

From (3), $a_B = \frac{(2)(29.294)}{5} - (0.20)(9.81) = 9.756 \text{ m/s}^2$

(*a*) Acceleration vectors. $\mathbf{a}_A = 6.83 \text{ m/s}^2 \longrightarrow$ ◀

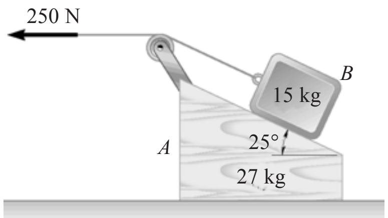
$\mathbf{a}_B = 9.76 \text{ m/s}^2 \longrightarrow$ ◀

$\mathbf{a}_C = 10 \text{ m/s}^2 \longrightarrow$ ◀

Since a_A, a_B, and a_C are to the right, the friction forces F_A, F_B, and F_C are to the left as assumed.

(*b*) Tension in the cable. $T = 29.3 \text{ N}$ ◀

(*c*) Force **P**. $\mathbf{P} = 237 \text{ N} \longrightarrow$ ◀



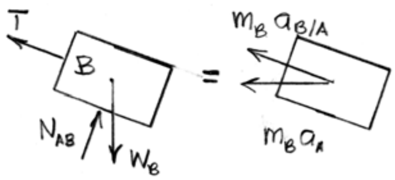
PROBLEM 12.30

The 15 kg block B is supported by the 27 kg block A and is attached to a cord to which a 250 N horizontal force is applied as shown. Neglecting friction, determine (a) the acceleration of block A, (b) the acceleration of block B relative to A.

SOLUTION

$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$, where $\mathbf{a}_{B/A}$ is directed along the inclined contact surface.

Block B: $+\nwarrow \Sigma F_x = \Sigma m a_x$:



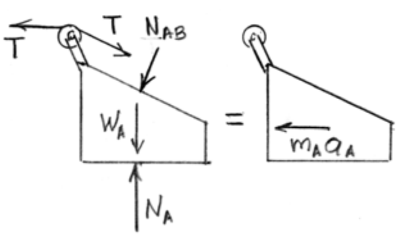
$$T - W_B \sin 25° = m_B a_A \cos 25° + m_B a_{B/A}$$

$$(15 \cos 25°)\ a_A + 15\ a_{B/A} = 250 - (15)\ (9.81)\ \sin 25°$$

$$13.595\ a_A + 15\ a_{B/A} = 187.812 \qquad (1)$$

$$+\nearrow \Sigma F_y = \Sigma m_B a_y: \quad N_{AB} - W_B \cos 25° = -m_B a_A \sin 25°$$

$$(15 \sin 25°)\ a_A + N_{AB} = (15)\ (9.81)\ \cos 25° \text{ or } 6.339\ a_A + N_{AB} = 133.363 \qquad (2)$$



Block A: $\overset{+}{\leftarrow} \Sigma F_x = m a_x$: $T - T\cos 25° + N_{AB} \sin 25° = m_A a_A$

$$27\ a_A - (\sin 25°)\ N_{AB} = 250\ (1 - \cos 25°)$$

$$27\ a_A - 0.42262 N_{AB} = 23.423 \qquad (3)$$

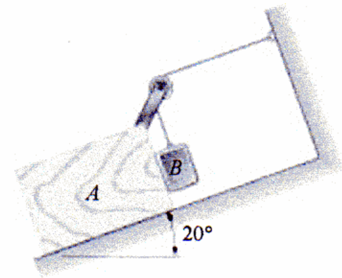
Using (2) and (3) to eliminate N_{AB} and solve for a_A,

(a) Acceleration of block A. $\mathbf{a}_A = 2.7\ \text{m/s}^2 \longleftarrow$ ◀

Substituting for a_A into (1) and solving for $a_{B/A}$,

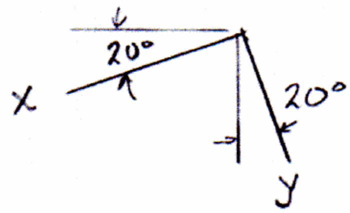
(b) Acceleration of B relative to A. $\mathbf{a}_{B/A} = 10\ \text{m/s}^2 \searrow 25°$ ◀

PROBLEM 12.32



A 25-kg block A rests on an inclined surface, and a 15-kg counterweight B is attached to a cable as shown. Neglecting friction, determine the acceleration of A and the tension in the cable immediately after the system is released from rest.

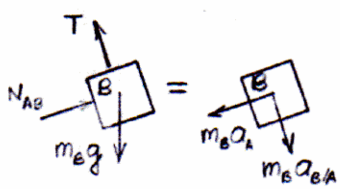
SOLUTION



Let the positive direction of x and y be those shown in the sketch, and let the origin lie at the cable anchor.

Constraint of cable: $x_A + y_{B/A} = \text{constant}$ or $a_A + a_{B/A} = 0$, where the positive directions of a_A and $a_{B/A}$ are respectively the x and the y directions. Then $a_{B/A} = -a_A$ (1)

First note that $\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A} = (a_A \nearrow 20°) + (a_{B/A} \searrow 20°)$



Block B: $+\!\swarrow \Sigma F_x = m_B (a_B)_x$: $m_B g \sin 20° - N_{AB} = m_B a_A$

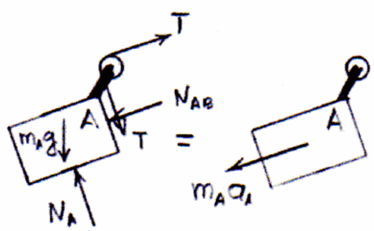
$$m_B a_A + N_{AB} = m_B g \sin 20°$$

$$15\, a_A + N_{AB} = 50.328 \quad (2)$$

$+\!\searrow \Sigma F_y = m_B (a_B)_y$: $m_B g \cos 20° - T = m_B a_{B/A}$

$$m_B a_{B/A} + T = m_B g \cos 20°$$

$$15\, a_{B/A} + T = 138.276 \quad (3)$$



Block A: $+\!\swarrow \Sigma F_x = m_A a_A$: $m_A g \sin 20° + N_{AB} - T = m_A a_A$

$$m_A a_A - N_{AB} + T = m_A g \sin 20°$$

$$25\, a_A - N_{AB} + T = 83.880 \quad (4)$$

Eliminate $a_{B/A}$ using Eq. (1), then add Eq. (4) to Eq. (2) and subtract Eq. (3).

$$55\, a_A = -4.068 \quad \text{or} \quad a_A = -0.0740 \text{ m/s}^2, \; \mathbf{a}_A = 0.0740 \text{ m/s}^2 \measuredangle \blacktriangleleft$$

From Eq. (1), $a_{B/A} = 0.0740 \text{ m/s}^2$

From Eq. (3), $T = 137.2$ N $\qquad T = 137.2 \text{ N} \blacktriangleleft$