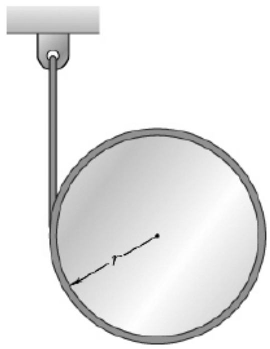
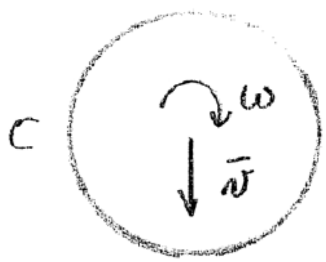
PROBLEM 17.21



A rope is wrapped around a cylinder of radius r and mass m as shown. Knowing that the cylinder is released from rest, determine the velocity of the center of the cylinder after it has moved downward a distance s.

SOLUTION



Point C is the instantaneous center.

$$\bar{v} = r\omega \qquad \omega = \frac{\bar{v}}{r}$$

Position 1. At rest. $T_1 = 0$

Position 2. Cylinder has fallen through distance s.

$$T_2 = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2 = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{\bar{v}}{r}\right)^2 = \frac{3}{4}m\bar{v}^2$$

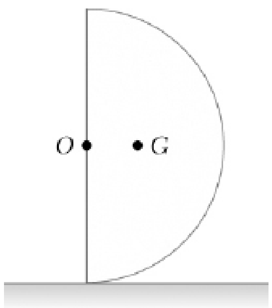
Work. $U_{1\to 2} = mgs$

Principle of Work and Energy.

$$T_1 + U_{1\to 2} = T_2: \quad 0 + mgs = \frac{3}{4}m\bar{v}^2$$

$$\bar{v}^2 = \frac{4gs}{3}$$

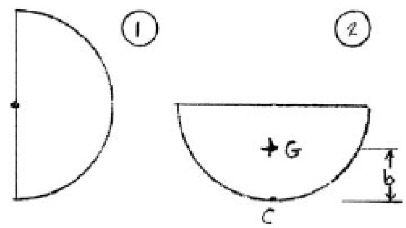
$$\bar{\mathbf{v}} = \sqrt{\frac{4gs}{3}} \downarrow \blacktriangleleft$$



PROBLEM 17.23

A half-cylinder of mass m and radius r is released from rest in the position shown. Knowing that the half-cylinder rolls without sliding, determine (*a*) its angular velocity after it has rolled through 90°, (*b*) the reaction at the horizontal surface at the same instant. [*Hint.* Note that $GO = 4r/3\pi$ and that, by the parallel-axis theorem, $\bar{I} = \frac{1}{2}mr^2 - m(GO)^2$.]

SOLUTION



Position 1. $T_1 = 0 \qquad V_1 = 0$

Position 2. $V_2 = -mg(OG) = -\frac{4}{3\pi}mgr$

Moments of inertia: $\bar{I} = I_0 - m(OG)^2$

$$\bar{I} = \frac{1}{2}mr^2 - m\left(\frac{4r}{3\pi}\right)^2 = 0.319873mr^2$$

Kinematics: Point C is the instantaneous center.

$$\bar{v} = v_G = b\omega_2 = \left(r - \frac{4r}{3\pi}\right)\omega_2 = 0.57559r\omega_2$$

Kinetic energy: $T_2 = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2 = \frac{1}{2}m(0.57559r\omega_2)^2 + \frac{1}{2}(0.319873)mr^2\omega_2^2$

$$= 0.32559mr^2\omega_2^2$$

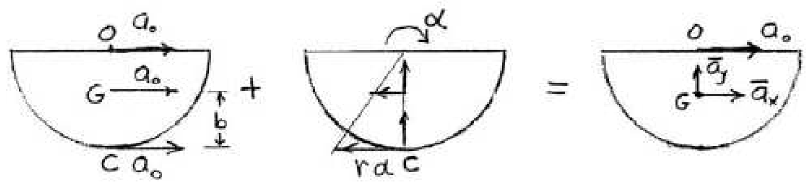
(*a*) Conservation of energy. $T_1 + V_1 = T_2 + V_2$:

$$0 + 0 = 0.32559mr^2\omega_2^2 - \frac{4}{3\pi}mgr \qquad \omega_2^2 = 1.3035\frac{g}{r}$$

$$\omega_2 = 1.142\sqrt{\frac{g}{r}} \;\circlearrowright \blacktriangleleft$$

PROBLEM 17.23 CONTINUED

Kinematics:

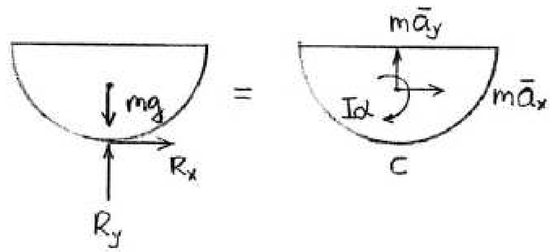


Translation + Rotation about O = Rolling Motion

(*b*) Acceleration of C. x component. $a_0 - r\alpha = 0 \qquad a_0 = r\alpha$

Acceleration of G. x component. $\bar{a}_x = a_0 - (OG)\alpha = b\alpha \longrightarrow$

y component. $\bar{a}_y = (OG)\omega_2^2 = \dfrac{4}{3\pi}r\omega_2^2 \uparrow$



$$+\circlearrowright \Sigma M_C = \Sigma(M_C)_{\text{eff}}:$$

$$0 = bm\bar{a}_x + \bar{I}\alpha$$

$$= mb^2\alpha + \bar{I}\alpha$$

$$\alpha = 0, \qquad \bar{a}_x = 0$$

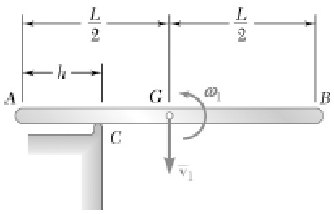
$$\xrightarrow{+} \Sigma F_x = m\bar{a}_x: \quad R_x = m\bar{a}_x \qquad R_x = 0$$

$$+\uparrow \Sigma F_y = m\bar{a}_y: \quad R_y - mg = m\left(\frac{4r}{3\pi}\right)\omega^2$$

$$R_y = mg + m\left(\frac{4r}{3\pi}\right)1.3035\frac{g}{r} \qquad R_y = 1.553mg$$

$\mathbf{R} = 1.553mg \uparrow$ ◀

PROBLEM 17.95



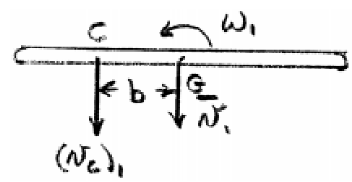
A uniform slender rod AB of mass m and length L strikes a rigid frictionless support at point C with an angular velocity of magnitude ω_1 when the velocity of its mass center G is zero $(v_1 = 0)$. Knowing that the angular velocity of the rod immediately after the impact is $\omega_1/2$ counterclockwise and assuming perfectly elastic impact, determine (*a*) the ratio h/L, (*b*) the velocity of the mass center of the rod immediately after the impact, (*c*) the impulse exerted on the rod at point C.

SOLUTION

Moment of inertia.

$$\bar{I} = \frac{1}{12}mL^2$$

Kinematics before impact. Let $b = \frac{L}{2} - h$.



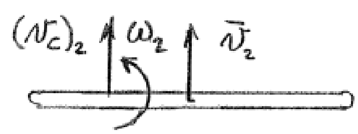
$$(v_C)_1 = \bar{v}_1 + b\omega = 0 + b\omega_1 = b\omega_1 \downarrow$$

Rebound at C.

$$e = 1$$

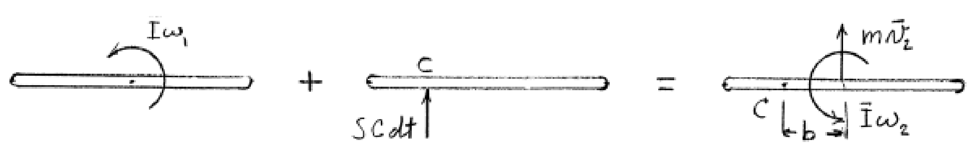
$$(v_C)_2 = (v_C)_1 \qquad (\mathbf{v}_C)_2 = b\omega_1 \uparrow$$

Kinematics after impact.



$$\bar{v}_2 = (v_C)_2 + b\omega_2 = b\omega_1 + b\left(\frac{1}{2}\omega_1\right) = \frac{3}{2}b\omega_1$$

Kinetics.



Syst Momenta$_1$ + **Syst Ext Imp**$_{1\to 2}$ = **Syst Momenta**$_2$

↻ moments about C:

$$\bar{I}\omega_1 + 0 = \bar{I}\omega_2 + m\bar{v}_2 b$$

$$\frac{1}{12}mL^2\omega_1 + 0 = \frac{1}{12}mL^2\left(\frac{1}{2}\omega_1\right) + m\left(\frac{3}{2}b\omega_1\right)(b) \qquad b = \frac{1}{6}L$$

PROBLEM 17.95 CONTINUED

(*a*)

$$h = \frac{L}{2} - b = \frac{L}{2} - \frac{L}{6}$$

$$\frac{h}{L} = \frac{1}{3} \blacktriangleleft$$

(*b*)

$$\bar{v}_2 = \left(\frac{3}{2}\right)\left(\frac{1}{6}L\right)\omega_1$$

$$\bar{\mathbf{v}}_2 = \frac{1}{4}\omega_1 L \uparrow \blacktriangleleft$$

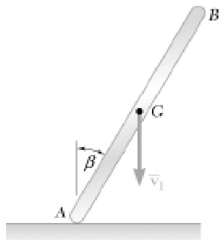
$+\uparrow$ vertical components:

$$0 + \int C dt = m\bar{v}_2$$

(*c*)

$$\int C dt = m\left(\frac{1}{4}\omega_1 L\right)$$

$$\int \mathbf{C} dt = \frac{1}{4}m\omega_1 L \uparrow \blacktriangleleft$$



PROBLEM 17.98

The slender rod AB of length L forms an angle β with the vertical axis as it strikes the frictionless surface shown with a vertical velocity $\bar{\mathbf{v}}_1$ and no angular velocity. Assuming that the impact is perfectly elastic, derive an expression for the angular velocity of the rod immediately after the impact.

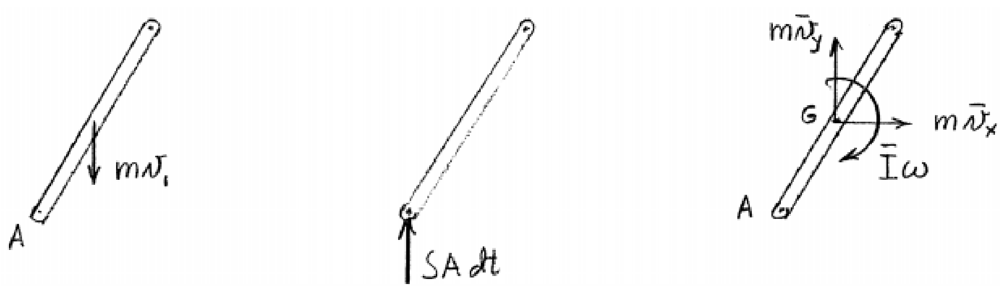
SOLUTION

Moment of inertia. $\bar{I} = \frac{1}{12}mL^2$

Perfectly elastic impact. $e = 1 \qquad \left[(v_A)_y\right]_2 = -e\left[(v_A)_y\right]_1 = ev_1 \uparrow$

$$\mathbf{v}_A = (v_A)_x\mathbf{i} + (v_A)_y\mathbf{j} = (v_A)_x\mathbf{i} + v_1\mathbf{j}$$

Kinetics.



Syst Momenta$_1$ + **Syst Ext Imp**$_{1\to 2}$ = **Syst Momenta**$_2$

$\xrightarrow{+}$ horizontal components: $0 + 0 = m\bar{v}_x \qquad m\bar{v}_x = 0$

Kinematics. $\mathbf{v}_G = \mathbf{v}_A + \mathbf{v}_{G/A} \qquad \left[\bar{v}_y \uparrow\right] = \left[v_1 \uparrow\right] + \left[(v_A)_x \longrightarrow\right] + \left[\frac{L}{2}\omega \nwarrow \beta\right]$

Velocity components $\uparrow$: $v_y = v_1 - \frac{L}{2}\omega\sin\beta$

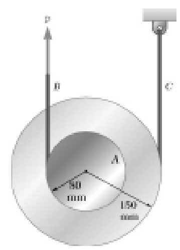
$\circlearrowright$ moments about A: $mv_1\frac{L}{2}\sin\beta + 0 = -m\bar{v}_y\frac{L}{2}\sin\beta + \bar{I}\omega$

$$mv_1\frac{L}{2}\sin\beta = m\left(\frac{L}{2}\omega\sin\beta - v_1\right)\frac{L}{2}\sin\beta + \frac{1}{12}mL^2\omega$$

$$\left(\frac{1}{12}mL^2 + \frac{1}{4}mL^2\sin^2\beta\right)\omega = mv_1L\sin\beta$$

$$\omega = \frac{12\sin\beta}{1+3\sin^2\beta}\frac{v_1}{L} \blacktriangleleft$$

PROBLEM 17.128

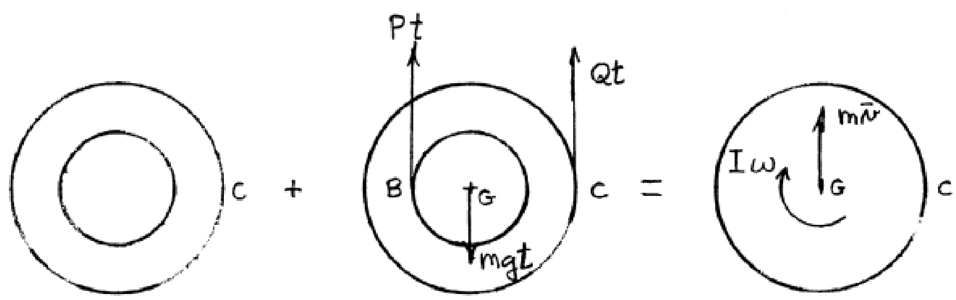


The double pulley shown has a mass of 3 kg and a radius of gyration of 100 mm. Knowing that when the pulley is at rest, a force **P** of magnitude 24 N is applied to cord *B*, determine (*a*) the velocity of the center of the pulley after 1.5 s, (*b*) the tension in cord *C*.

SOLUTION

For the double pulley, $r_C = 0.150 \text{ m}, r_B = 0.080 \text{ m}, k = 0.100 \text{ m}$

Principle of Impulse and Momentum.



Syst Momenta$_1$ + **Syst Ext Imp**$_{1\to2}$ = **Syst Momenta**$_2$

Kinematics. Point *C* is the instantaneous center. $\bar{v} = r_C\omega$

↶ moments about *C*: $0 + Pt(r_C + r_B) - mgtr_C = \bar{I}\omega + m\bar{v}r_C$

$$= mk^2\omega + m(r_C\omega)r_C$$

$$\omega = \frac{Pt(r_C + r_B) - mgtr_C}{m(k^2 + r_C^2)} = \frac{(24)(1.5)(0.230) - (3)(9.81)(1.5)(0.150)}{3(0.100^2 + 0.150^2)}$$

$$= 17.0077 \text{ rad/s}$$

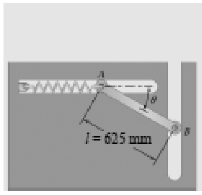
(*a*) $\bar{v} = (0.150)(17.0077) = 2.55115 \text{ m/s}$

$\bar{v} = 2.55 \text{ m/s} \uparrow$ ◀

$+\uparrow$ linear components: $0 + Pt - mgt + Qt = m\bar{v}$

$$Q = \frac{m\bar{v}}{t} + mg - P = \frac{(3)(2.55115)}{1.5} + (3)(9.81) - 24$$

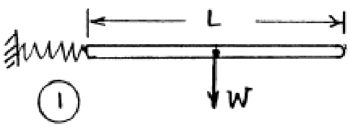
(*b*) *Tension in cord C.* $Q = 10.53 \text{ N}$ ◀



PROBLEM 17.132

The ends of a 4 kg rod AB are constrained to move along slots cut in a vertical plane as shown. A spring of constant $k = 550$ N/m is attached to end A in such a way that its tension is zero when $\theta = 0$. If the rod is released from rest when $\theta = 0$, determine the angular velocity of the rod and the velocity of end B when $\theta = 30°$.

SOLUTION



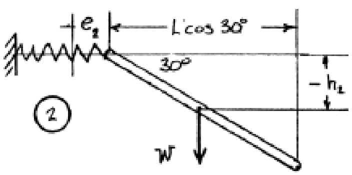
Moment of inertia. Rod. $\bar{I} = \frac{1}{12}mL^2$

Position 1. $\theta_1 = 0 \qquad \bar{v}_1 = 0 \qquad \omega_1 = 0$

h_1 = elevation above slot. $h_1 = 0$

e_1 = elongation of spring. $e_1 = 0$

$$T_1 = \frac{1}{2}m\bar{v}_1^2 + \frac{1}{2}\bar{I}\omega_1^2 = 0$$



$$V_1 = \frac{1}{2}ke_1^2 + mgh_1 = 0$$

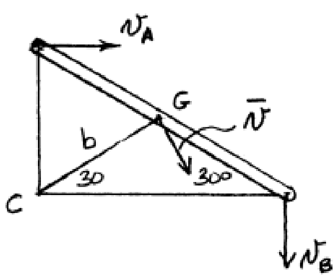
Position 2 $\theta = 30°$

$$e_2 + L\cos 30° = L \qquad e_2 = L(1 - \cos 30°)$$

$$h_2 = -\frac{L}{2}\sin 30° = -\frac{1}{4}L$$

$$V_2 = \frac{1}{2}ke_2^2 + mgh_2$$

$$= \frac{1}{2}kL^2(1 - \cos 30°)^2 - \frac{1}{4}mgL$$



Kinematics. Velocities at A and B are directed as shown. Point C is the instantaneous center of rotation. From geometry, $b = \frac{L}{2}$.

$$\bar{v} = b\omega = \frac{L}{2}\omega \qquad v_B = (L\cos 30°)\omega$$

$$T_2 = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2 = \frac{1}{2}m\left(\frac{L}{2}\omega\right)^2 + \frac{1}{2}\left(\frac{1}{12}mL^2\right) = \frac{1}{6}mL^2\omega^2$$

Conservation of energy. $T_1 + V_1 = T_2 + V_2$:

$$0 + 0 = \frac{1}{6}mL^2\omega^2 + \frac{1}{2}kL^2(1 - \cos 30°)^2 - \frac{1}{4}mgL$$

$$\omega^2 = \frac{3g}{2L} - \frac{3k}{m}\ (1 - \cos 30°)^2$$

PROBLEM 17.132 CONTINUED

Data: $m = 4$ kg, $g = 9.81$ m/s^2, $L = 625$ mm $= 0.625$ m

$k = 550$ N/m

$$\omega^2 = \frac{(3)(9.81)}{2 \times 0.625} - \frac{(3)(550)(1-\cos 30°)^2}{4} = 39.684$$

$\omega = 4.018$ rad/s ↻ ◀

$$v_B = (0.625)\ (\cos\ 30°)(4.018)$$

$v_B = 2.175$ m/s ↓ ◀