LAST NAME SOLUTIONS

FIRST NAME _____

STUDENT NO. _____

Department of Civil Engineering and Applied Mechanics
McGill University

ANALYTICAL MECHANICS, CIVE 281

Test No. 2

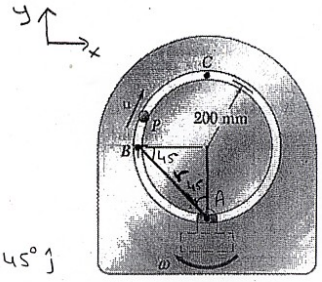
Examiner: Prof. S. Babarutsi
Prof. V. H. Chu

Date: Monday, November 12, 2007
Time: 8:30 a.m. - 9:25 a.m.

Answer both questions on the space provided below the question. Continue on the facing page if more space is required. Both questions are of equal value. Textbooks and lecture notes are not permitted.

QUESTION	MARK
1. (50%)	
2. (50%)	
TOTAL	

1. (50%) Pin P slides in a circular slot cut in the plate shown below at a constant relative speed $u = 180$ mm/s. Knowing that at the instant shown the plate rotates clockwise about A at the constant rate $\omega = 6$ rad/s, determine the acceleration of the pin if it is located at point B.



$$\text{velocity} = \frac{dR_o}{dt} + \omega \times \mathbf{r} + \dot{\mathbf{r}}$$

$$\text{acceleration} = \frac{d^2R_o}{dt^2} + \dot{\omega} \times \mathbf{r} + \omega \times (\omega \times \mathbf{r}) + 2\omega \times \dot{\mathbf{r}} + \ddot{\mathbf{r}}$$

$\underline{\omega} = -6\hat{k}$ $\quad \underline{\dot{\omega}} = 0$

$\underline{r} = -r\cos 45^\circ \hat{i} + r\sin 45^\circ \hat{j} = -0.2\sqrt{2}\cos 45^\circ \hat{i} + 0.2\sqrt{2}\sin 45^\circ \hat{j}$

$\underline{r} = -0.2\hat{i} + 0.2\hat{j}$

$a_A = 0$

$\underline{\dot{r}} = 0.18\hat{j}$

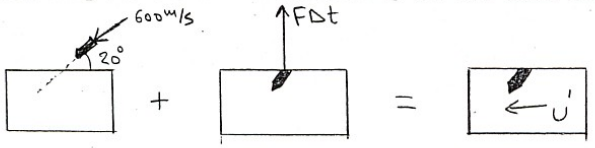
$\underline{\ddot{r}} = \frac{u^2}{\rho} = \frac{(0.18)^2}{0.2}\hat{i} = 0.162\hat{i}$ m/s^2

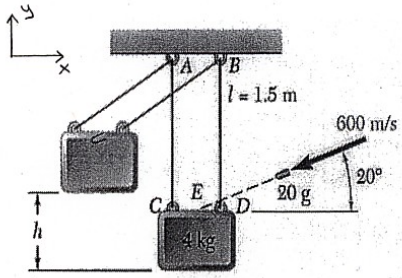
$\underline{\omega} \times (\underline{\omega} \times \underline{r}) = -6\hat{k} \times (-6\hat{k} \times (-0.2\hat{i} + 0.2\hat{j})) = 7.2\hat{i} - 7.2\hat{j}$

$2\underline{\omega} \times \dot{r} = 2(-6\hat{k}) \times 0.18\hat{j} = 2.16\hat{i}$

$a_P = 0 + 0 + 7.2\hat{i} - 7.2\hat{j} + 2.16\hat{i} + 0.162\hat{i} = \boxed{9.522\hat{i} - 7.2\hat{j}}$

2. (50%) A 20 g bullet fired into a 4 kg wooden block suspended from cords AC and BD penetrates the block at point E, halfway between C and D, without hitting cord BD. Determine (a) the maximum height h to which the block and the embedded bullet will swing after impact, (b) the total impulse exerted on the block by the two cords during the impact.





$m_{BL}\underline{v}_{BL} + m_{BU}\underline{v}_{BU} + \underline{F}\Delta t = (m_{BL}+m_{BU})\underline{v}'$

x-direction:

$0 - m_{BU} v_{BU} \cos 20^\circ + 0 = -(m_{BL}+m_{BU})v'$

$-0.02(600\,m/s)\cos 20^\circ = -v'(4+0.02kg) \Rightarrow v' = 2.8\,m/s$

Conservation of Energy:

$T_1 = \frac{1}{2}(m_{BL}+m_{BU})v'^2 = \frac{1}{2}(4+0.02kg)(2.8)^2 = 15.76\,J$

$V_1 = 0 \qquad T_2 = 0$

$V_2 = (m_{BL}+m_{BU})gh = 4.02kg(9.81)h = 39.44h$

$T_1+V_1 = T_2+V_2 \Rightarrow 15.76+0 = 0+39.44h \Rightarrow \boxed{h = 0.4\,m}$

y-direction

$0 - 0.02(600\,m/s)\sin 20^\circ + F\Delta t = 0$

$\boxed{F\Delta t = 4.10\,Ns}$