LAST NAME ______________________________

FIRST NAME ______________________________

STUDENT NO. ______________________________

Department of Civil Engineering and Applied Mechanics
McGill University

CIVE 281 ANALYTICAL MECHANICS
Final Examination

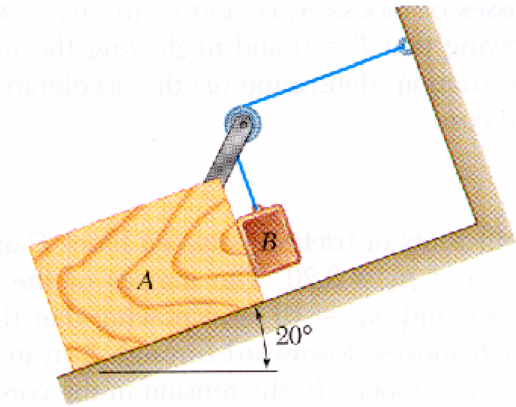
Examiners: Professor V. H. Chu
Professor S. Babarutsi

Date: Thursday, December 7, 2006
Time: 2:00 p.m. - 5:00 p.m.

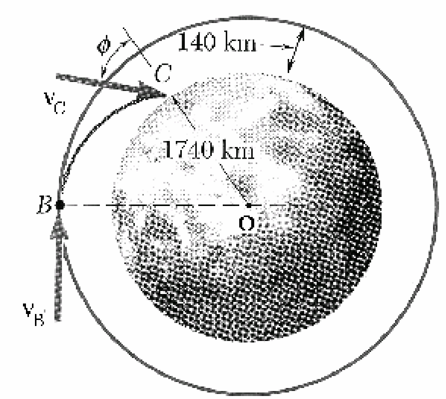
This exam consists of six questions on 8 pages. Answer **FIVE** of the six questions on the sheets provided. All questions are of equal value.

If more space is required, use the back of the sheets or the blank page at the end. One sheet ($8\frac{1}{2}$" $\times$ 11" both sides) of your own hand-written notes may be used. Textbooks and lecture notes are not permitted.

1. A 25-kg block A rests on an inclined surface, and a 15-kg counterweight B is attached to a cable as shown. Neglecting friction, determine the acceleration of block A and the tension in the cable immediately after the system is released from rest.



2. Upon the lunar excursion module's (LEM) return to the command module, the Apollo spacecraft was turned around so that the LEM faced to the rear. At point B, the LEM was then cast adrift with a velocity of 180 m/s relative to the command module which was orbiting the moon at the altitude of 140 km. Determine the magnitude and direction (angle Φ formed with the vertical OC) of the velocity $\mathbf{v}_C$ of the LEM just before it crashed at C on the moon's surface. The radius of the moon is 1740 km and its mass is 0.01230 times the mass of the earth. The radius of the earth is 6370 km.



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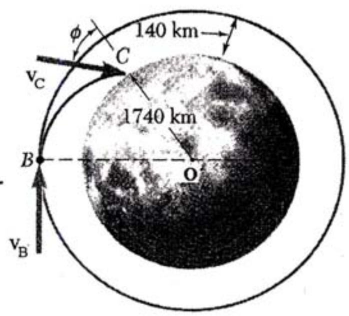


$r_B = 1740\text{ km} + 140\text{ km} = 1880\text{ km} = 1.88 \times 10^6\text{ m}$

$GM_{moon} = 0.01230\, GM_{earth} = 0.0123\, gR^2$

$= 0.0123(9.81\text{ m/s}^2)(6.37\times10^6\text{ m})^2 = 4.896\times10^{12}\text{ m}^3/\text{s}^2$

Command module:

$$v_{cir} = \sqrt{\frac{GM_{moon}}{r_B}} = \sqrt{\frac{4.896\times10^{12}}{1.88\times10^6}} = 1613.8\text{ m/s}$$

$v_B = 1613.8\text{ m/s} - 180\text{ m/s} = 1434\text{ m/s}$

Conservation of energy between B and C:

$$\frac{1}{2}mv_B^2 - \frac{GM_{moon}m}{r_B} = \frac{1}{2}mv_c^2 - \frac{GM_{moon}m}{r_c}$$

$$v_c^2 = v_B^2 + \frac{2GM_{moon}}{r_B}\left(\frac{r_B}{r_c} - 1\right) = (1434)^2 + 2\,\frac{4.896\times10^{12}}{1.88\times10^6}\left(\frac{1.88\times10^6}{1.74\times10^6} - 1\right)$$

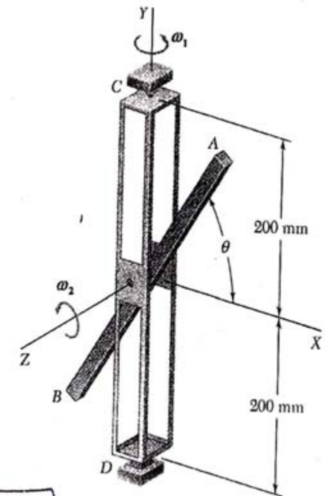
$$\boxed{v_c = 1573.35\text{ m/s}}$$

Conservation of angular momentum

$$r_B m v_B = R m v_c \sin\phi \Rightarrow \sin\phi = \frac{r_B v_B}{R v_c} = \frac{1.88\times10^6\text{ m}\times1434}{1.740\times10^6\times1573.35}$$

$$\sin\phi = 0.985 \Rightarrow \boxed{\phi = 79.98^\circ}$$

3. The 400-mm bar AB is made to rotate at the rate $\omega_2 = d\theta/dt$ with respect to the frame CD which itself rotates at the rate ω_1 about the Y axis. At the instant shown $\omega_1 = 12$ rad/s, $d\omega_1/dt =$ -16 rad/s^2, $\omega_2 = 8$ rad/s, $d\omega_2/dt = 10$ rad/s^2, and $\theta = 60^o$. Determine the velocity and acceleration of point A at this instant.



$$\text{velocity} = \frac{dR}{dt} + \omega \times \mathbf{r} + \dot{\mathbf{r}}$$

$$\text{accelaration} = \frac{d^2R}{dt^2} + \dot{\omega} \times \mathbf{r} + \omega \times (\omega \times \mathbf{r}) + \mathbf{2}\omega \times \dot{\mathbf{r}} + \ddot{\mathbf{r}}$$

0.5 0.5 1 1

$$\omega_1 = 12\hat{j} \quad \omega_2 = 8\hat{k} \quad \dot{\omega}_1 = \frac{d\omega_1}{dt} = -16\hat{j} \quad \dot{\omega}_2 = \frac{d\omega_2}{dt} = 10\hat{k}$$

$$r = r_A = 0.2\text{m}(\cos 60^\circ \hat{i} + \sin 60^\circ \hat{j}) = 0.1\hat{i} + 0.1732\hat{j}$$

$$\dot{r} = \omega_2\hat{k} \times r = 8\hat{k} \times (0.1\hat{i} + 0.1732\hat{j}) = 0.8\hat{j} - 1.3856\hat{i}$$

$$V_A = 12\hat{j} \times (0.1\hat{i} + 0.1732\hat{j}) + 0.8\hat{j} - 1.3856\hat{i}$$

$$V_A = -1.2\hat{k} + 0 + 0.8\hat{j} - 1.3856\hat{i} = \boxed{-1.3856\hat{i} + 0.8\hat{j} - 1.2\hat{k}}$$

$$\ddot{r} = \dot{\omega}_2\hat{k} \times r + \omega_2\hat{k} \times \dot{r} = 10\hat{k} \times (0.1\hat{i} + 0.1732\hat{j}) + 8\hat{k} \times (0.8\hat{j} - 1.3856\hat{i})$$

$$\ddot{r} = 1\hat{j} - 1.732\hat{i} - 6.4\hat{i} - 11.084\hat{j} = -8.132\hat{i} - 10.084\hat{j}$$

$$a_A = -16\hat{j} \times (0.1\hat{i} + 0.1732\hat{j}) + 12\hat{j} \times \left(12\hat{j} \times (0.1\hat{i} + 0.1732\hat{j})\right) + 2(12\hat{j}) \times (0.8\hat{j} - 1.3856\hat{i})$$
$$+ (-8.132\hat{i} - 10.084\hat{j})$$

$$a_A = -16\hat{j} \times (0.1\hat{i} + 0.1732\hat{j}) + 12\hat{j} \times [-1.2\hat{k}] + 2(12\hat{j}) \times (0.8\hat{j} - 1.3856\hat{i}) + (-8.132\hat{i} - 10.084$$

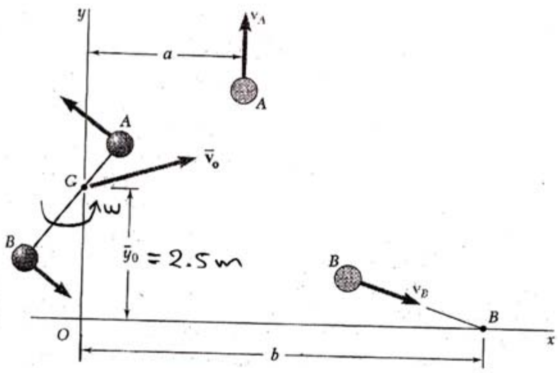
$$a_A = 1.6\hat{k} + 0 - 14.4\hat{i} + 2(0 + 16.62\hat{k}) - 8.132\hat{i} - 10.084\hat{j}$$

$$a_A = 1.6\hat{k} - 14.4\hat{i} + 33.25\hat{k} - 8.132\hat{i} - 10.084\hat{j}$$

$$\boxed{a_A = -22.532\hat{i} - 10.084\hat{j} + 34.85\hat{k}}$$

4. Two small disks, A and B, of mass 3 kg and 1.5 kg, respectively, can slide on a horizontal and frictionless surface. They are connected by a cord of negligible mass and spin about their mass center G. At $t = 0$, G is moving with the velocity $\overline{\mathbf{v}}_{\mathbf{o}}$ and its coordinates are $\overline{x}_o = 0$, $\overline{y}_o = 2.5$ m. Shortly thereafter, the cord breaks and disk A is observed to move with a velocity $\overline{\mathbf{v}}_{\mathbf{A}} = (2.56 \text{ m/s})$ **j** in a straight line and at a distance $a = 1.861$ m from the y axis, while B moves with a velocity $\overline{\mathbf{v}}_{\mathbf{B}} = (3.6 \text{ m/s})$ **i** - (2.24 m/s) **j** along a path intersecting the x axis at a distance $b = 7.2$ m from the origin O. Determine (a) the initial velocity $\overline{\mathbf{v}}_{\mathbf{o}}$ of the mass center G of the two disks, (b) the length of the cord initially connecting the two disks, (c) the rate in rad/s at which the disks were spinning about G. ($\mathbf{H_O} = \mathbf{H_G} + \mathbf{r_G} \times \text{m } \mathbf{v_G}$).

$AG = \frac{1}{3}\ell \qquad BG = \frac{2}{3}\ell$



Conservation of Linear Momentum

$(m_A + m_B)\bar{V}_0 = m_A V_A + m_B V_B$

$4.5\bar{V}_0 = 3(2.56)\hat{j} + 1.5(3.6\hat{i} - 2.24\hat{j})$

$\bar{V}_0 = 1.706\hat{j} + 1.2\hat{i} - 0.7466\hat{j} = \boxed{1.2\hat{i} + 0.96\hat{j}}$

$V_{A/G} = \frac{1}{3}\ell\omega \qquad V_{B/G} = \frac{2}{3}\ell\omega$

Initial kinetic energy:

$T_0 = \frac{1}{2}(m_A + m_B)\bar{V}_0^2 + \frac{1}{2}m_A\left(\frac{1}{3}\ell\omega\right)^2 + \frac{1}{2}m_B\left(\frac{2}{3}\ell\omega\right)^2 = \frac{1}{2}(4.5)(1.2^2 + 0.96^2) + \frac{1}{2}(3)\left(\frac{1}{9}\ell^2\omega^2\right) + \frac{1}{2}(1.5)\left(\frac{4}{9}\ell^2\omega^2\right)$

$T_0 = 5.3136 + 0.1666\ell^2\omega^2 + 0.333\ell^2\omega^2 = 5.3136 + 0.5\ell^2\omega^2$

Final kinetic energy:

$T = \frac{1}{2}m_A V_A^2 + \frac{1}{2}m_B V_B^2 = \frac{1}{2}(3)(2.56)^2 + \frac{1}{2}(1.5)(3.6^2 + 2.24^2) = 9.83 + 13.4832 = 23.3132 \text{ kg m}^2/\text{s}^2$

Conservation of energy:

$T_0 = T \Rightarrow 5.3136 + 0.5\ell^2\omega^2 = 23.3132 \Rightarrow \ell^2\omega^2 = 36 \Rightarrow \ell\omega = 6 \text{ m/s} \quad (1)$

Conservation of angular momentum about O:

$(H_0)_1 = H_G + r_G \times mV_G = \left(\frac{1}{3}\ell\right)(3\text{kg})\left(\frac{1}{3}\ell\omega\right)\hat{k} + \left(\frac{2}{3}\ell\right)(1.5\text{kg})\left(\frac{2}{3}\ell\omega\right)\hat{k} + (2.5\hat{j}) \times (4.5\text{kg})(1.2\hat{i} + 0.96\hat{j})$

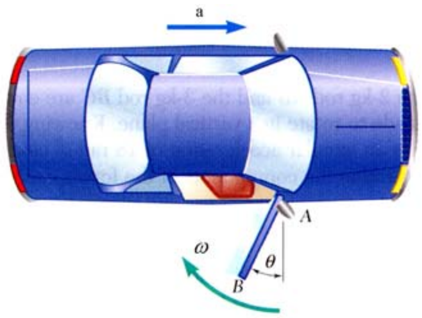
$= \frac{1}{3}\ell^2\omega\hat{k} + \frac{2}{3}\ell^2\omega\hat{k} + (2.5\hat{j}) \times (5.4\hat{i} + 4.32\hat{j}) = \ell^2\omega\hat{k} - 13.5\hat{k} = \ell^2\frac{6}{\ell}\hat{k} - 13.5\hat{k} = 6\ell\hat{k} - 13.5\hat{k}$

$(H_0)_2 = 1.861\hat{i} \times (3\text{kg})\, 2.56\hat{j} + 7.2\hat{i} \times (1.5\text{kg})(3.6\hat{i} - 2.24\hat{j}) = 14.29\hat{k} - 24.192\hat{k} = -9.902\hat{k}$

$(H_0)_1 = (H_0)_2 \Rightarrow 6\ell\hat{k} - 13.5\hat{k} = -9.902\hat{k} \Rightarrow 6\ell\hat{k} = 3.598\hat{k} \Rightarrow \boxed{\ell = 0.6 \text{ m}}$

Eq. (1) $\Rightarrow \ell\omega = 6 \Rightarrow \omega = \frac{6}{\ell} = \frac{6}{0.6} = \boxed{10 \text{ rad/s}}$

5. A driver starts his car with the door on the passenger's side wide open ($\theta = 0$). The 30-kg door has a centroidal radius of gyration of $r_g = 0.25$ m, and its mass center located at a distance of 0.50 m from its vertical axis of rotation. Knowing that the driver maintains a constant of acceleration of 1.5 m/s^2, determine the angular velocity of the door as it slams shut ($\theta = 90^o$). Neglect friction at the hinge. Hint: $\overline{I} = mr_g^2$



6. A half-cylinder of mass $m = 10$ kg and radius $r = 1$ m is released from rest in the position shown. Knowing that the half-cylinder rolls withour sliding, determine (a) its angular velocity after it has rolled through 90^o, and (b) the force of the reaction at the horizontal surface at the same instant. [Hint: Note that GO $= 4r/3\pi$ and that, by the parallel-axis theorem, $\overline{I} = \frac{1}{2}mr^2 - m(\mathrm{GO})^2$.]