LAST NAME SOLUTIONS

FIRST NAME ____________________

STUDENT NO. ____________________

Department of Civil Engineering and Applied Mechanics
McGill University

ANALYTICAL MECHANICS, CIVE 281

Test No. 2

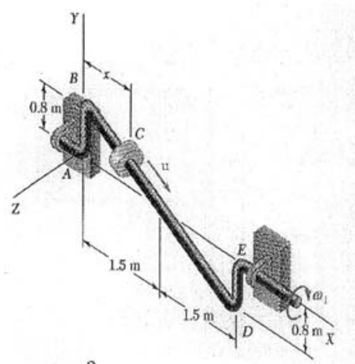
Examiner: Prof. S. Babarutsi, Prof. V. H. Chu

Date: Monday, November 13, 2006
Time: 8:30 a.m. - 9:25 a.m.

Answer both questions on the space provided below the question. Continue on the facing page if more space is required. Both questions are of equal value. Textbooks and lecture notes are not permitted.

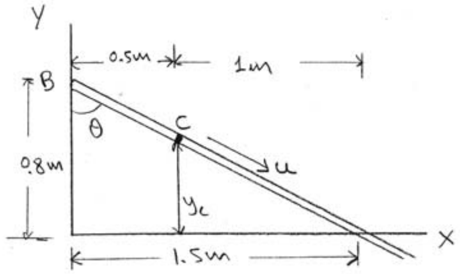
QUESTION	MARK
1. (50%)	
2. (50%)	
TOTAL	

1. (50%) The bent rod shown rotates at the constant rate $\omega_1 = 3$ rad/s. Knowing that collar C moves toward point D at a constant relative speed $u = 3.4$ m/s, determine, for the position shown, the velocity and acceleration of C if $x = 0.5$ m.



$$\text{velocity} = \frac{dR_o}{dt} + \omega \times \mathbf{r} + \dot{\mathbf{r}}$$

$$\text{acceleration} = \frac{d^2R_o}{dt^2} + \dot{\omega} \times \mathbf{r} + \omega \times (\omega \times \mathbf{r}) + \mathbf{2}\omega \times \dot{\mathbf{r}} + \ddot{\mathbf{r}}$$



$\frac{y_c}{0.8} = \frac{1m}{1.5m}$

$y_c = 0.533\,m$

$\theta = \tan^{-1}\frac{1.5}{0.8} = 61.9275°$

$r_{C/A} = 0.5\hat{i} + 0.533\hat{j}$ (10)

$\underline{\omega} = -3\,rad/s\,\hat{i}$

$u = 3.4\,m/s\,(\sin 61.927°)\hat{i} - 3.4\cos(61.9275)\hat{j} = 3\,m/s\,\hat{i} - 1.6\,m/s\,\hat{j}$

$V_c = 0 + (-3\hat{i}) \times (0.5\hat{i} + 0.533\hat{j}) + 3\hat{i} - 1.6\hat{j} = -1.6\hat{k} + 3\hat{i} - 1.6\hat{j}$

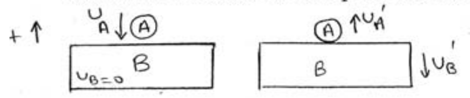
$\boxed{V_c = 3\hat{i} - 1.6\hat{j} - 1.6\hat{k}}$

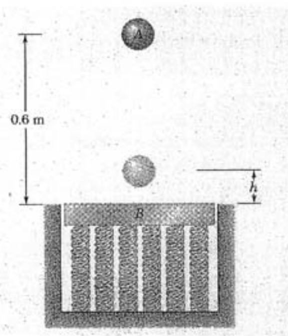
$a_c = 0 + 0 + (-3\hat{i}) \times (-3\hat{i}) \times (0.5\hat{i} + 0.533\hat{j}) + 2(-3\hat{i}) \times (3\hat{i} - 1.6\hat{j}) + 0$

$a_c = (-3\hat{i}) \times (-1.6\hat{k}) + 2(4.8\hat{k}) = -4.8\hat{j} + 9.6\hat{k}$

$\boxed{a_c = -4.8\hat{j} + 9.6\hat{k}}$

2. (50%) A 0.5 kg sphere A is dropped from a height of 0.6 m onto a 1 kg plate B, which is supported by a nested set of springs and is initially at rest. Knowing that the coefficient of restitution between the sphere and the plate is $e = 0.8$, determine (a) the height h reached by the sphere after rebound, (b) the constant k of the single spring equivalent to the given set if the maximum deflection of the plate is observed to be equal to $3h$.





Conservation of energy:

$T_1 = 0 \quad V_1 = m_A g(0.6\text{m}) \quad T_2 = \frac{1}{2} m_A v_A^2 \quad V_2 = 0$

$T_1 + V_1 = T_2 + V_2$

$0 + 0.6 m_A g = \frac{1}{2} m_A v_A^2 \Rightarrow v_A^2 = 2(0.6)(9.81\,\text{m/s}^2) = 11.772\,\text{m}^2/\text{s}^2$

$v_A = 3.431\ \text{m/s}$

Conservation of momentum

$m_A(-v_A) + m_B v_B = m_A v_A' + m_B(-v_B')$

$0.5(-3.431) + 0 = 0.5 v_A' + 1(-v_B') \Rightarrow -3.431 = v_A' - 2v_B' \quad (1)$

Coefficient of restitution:

$-v_B' - v_A' = e(-v_A - 0) \Rightarrow -v_B' - v_A' = 0.8(-3.431) \Rightarrow v_B' + v_A' = 2.7448\,\text{m/s} \quad (2)$

Solving (1) and (2)

$v_B' = 2.06\ \text{m/s} \qquad v_A' = 0.69\ \text{m/s}$

Conservation of energy for A:

$T_2 = \frac{1}{2} m_A v_A'^2 \quad V_2 = 0 \qquad T_3 = 0 \quad V_3 = m_A g h$

$T_2 + V_2 = T_3 + V_3 \Rightarrow \frac{1}{2} m_A v_A'^2 + 0 = 0 + m_A g h \Rightarrow h = \frac{v_A'^2}{2g} = \frac{(0.69)^2}{2(9.81)} = \boxed{0.02426\text{m} = 24.26\text{mm}}$

Conservation of energy:

$T_2 = \frac{1}{2} m_B v_B'^2 = \frac{1}{2}(1\text{kg})(2.06)^2 = 2.1218$

Initial deformation $= x = \frac{W_B}{k} = \frac{1\text{kg}(9.81)}{k} = \frac{9.81}{k}$

$V_2 = V_e = \frac{1}{2} k x^2 = \frac{1}{2} k \left(\frac{9.81}{k}\right)^2 = \frac{48.11}{k}$

$T_4 = 0$

$V_4 = V_g + V_e = m_B g(-h_4) + \frac{1}{2} k x_4^2 = -1\text{kg}(9.81)(3h - x) + \frac{1}{2} k (3h)^2 = -9.81\left(3h - \frac{9.81}{k}\right) + \frac{1}{2} k (3h)^2$

$V_4 = -0.7139 + \frac{96.23}{k} + 0.0026448k$

$T_2 + V_2 = T_4 + V_4$

$2.1218 + \frac{48.11}{k} = 0 - 0.7139 + \frac{96.23}{k} + 0.0026448k \Rightarrow 2.8357 = \frac{48.112}{k} + 0.002648k$

Solve for k: $\boxed{k = 1054\ \text{N/m}}$