LAST NAME Solution

FIRST NAME _____

STUDENT NO. _____

Department of Civil Engineering and Applied Mechanics
McGill University

CIVE281 ANALYTICAL MECHANICS

Test No.1

Examiners: Prof. V. H. Chu
Prof. S. Babarutsi

Date: Wednesday, October 11, 2006
Time: 8:30 a.m. - 9:25 a.m.

Answer on the space provided below the question. Continue on the facing page if more space is needed.

QUESTION	MARK
1 (40%)	
2 (60%)	
TOTAL	

1. (40%) A rocket is tracked by radar from its launching point A. When it is 10 s into its flight, the following radar measurements are recorded: $r = 1000$ m, $\dot{r} = 300$ m/s, $\ddot{r} = 5$ m/s^2, $\theta = 30^o$, $\dot{\theta} = 0.08$ rad/s, $\ddot{\theta} = -0.03$ rad/s^2. Determine the angle β between the horizontal and the direction of the trajectory of the rocket. Hint: $\hat{\mathbf{j}} = (\sin\theta)\hat{\mathbf{e}}_r + (\cos\theta)\hat{\mathbf{e}}_\theta$

$v_r = \dot{r} = 300$ m/s

$v_\theta = r\dot{\theta} = 1000 \times 0.08 = 80$ m/s

$v = \sqrt{v_r^2 + v_\theta^2} = 310.5$ m/s

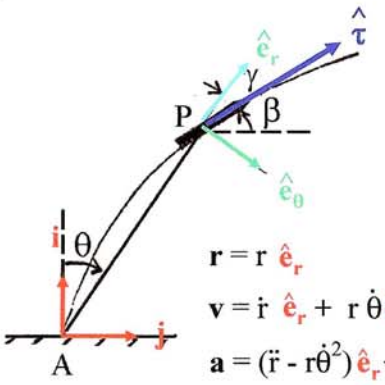
$\hat{\tau} = \frac{300}{310.5}\hat{e}_r + \frac{80}{310.5}\hat{e}_\theta$

$= 0.9662\,\hat{e}_r + 0.2576\,\hat{e}_\theta$

$\hat{\tau} \cdot \hat{j} = \cos\beta = (.9662, .2576) \cdot (0.5, 0.866)$

$= 0.7062$

$\beta = 45.07^\circ$



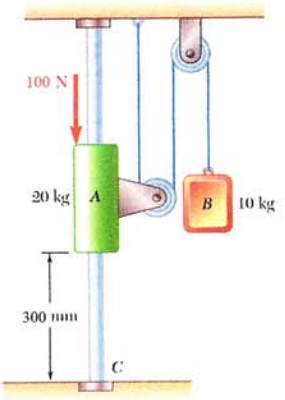
$\mathbf{r} = r\,\hat{\mathbf{e}}_r$

$\mathbf{v} = \dot{r}\,\hat{\mathbf{e}}_r + r\dot{\theta}\,\hat{\mathbf{e}}_\theta$

$\mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\,\hat{\mathbf{e}}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\,\hat{\mathbf{e}}_\theta$

2. (60%) The system shown, consisting of a 20-kg collar A and a 10-kg counterweight B, is at rest when a constant 100-N force is applied to collar A. Neglect friction and mass of the pulleys. Determine (a) the velocity of block B and (b) the tension in the cable just before block A hits the support at C.

$$\int_1^2 \vec{F}_n \cdot d\vec{r} = (T_2 + V_2) - (T_1 + V_1)$$



(a) $100 \times 0.3 = \frac{1}{2} \cdot 20 v_A^2 + \frac{1}{2} \cdot 10 v_B^2$

$- 20g(0.3) + 10g(0.6)$

$= 7.5 v_B^2$

$v_B = \sqrt{\frac{100 \times 0.3}{7.5}} = 2 \text{ m/s}.$

Kinematics:

0.3 m down at A

0.6 m up at B

$v_B = 2 v_A$

(b)

$T \times 0.6 = \frac{1}{2} 10 v_B^2 + 10g(0.6)$

$T = \frac{20 + 6g}{0.6} = 131.4 \text{ N}$