LAST NAME ______________________________

FIRST NAME ______________________________

STUDENT NO. ______________________________

Department of Civil Engineering and Applied Mechanics
McGill University

CIVE 281 ANALYTICAL MECHANICS
Final Examination

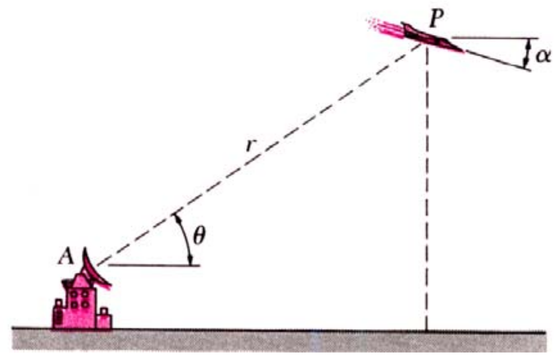
Examiners: Professor V. H. Chu
Professor S. Babarutsi

Date: Wednesday, December 7, 2005
Time: 9:00 a.m. - 12:00 noon

This exam consists of six questions on 8 pages. Answer **FIVE** of the six questions on the sheets provided. All questions are of equal value.

If more space is required, use the back of the sheets or the blank page at the end. One sheet ($8\frac{1}{2}$" $\times$ 11" both sides) of your own hand-written notes may be used. Textbooks and lecture notes are not permitted.

1. An airplane is tracked by radar. When the airplane is at P, the following radar measurements are recorded: $r = 3800$ m, $\dot{r} = 150$ m/s, $\ddot{r} = 4.10$ m/s^2, $\theta = 30^o$C, $\dot{\theta} = -\ 0.052$ rad/s, $\ddot{\theta} = -\ 0.011$ rad/s^2. Determine (a) the magnitude of the plane's speed v, (b) the magnitude of the plane's acceleration a, and (c) the angle of dive α of the plane at that position. Hint: $v_r = \dot{r}$, $v_\theta = r\,\dot{\theta}$, $a_r = \ddot{r} - r\,\dot{\theta}^2$, $a_\theta = r\,\ddot{\theta} + 2\,\dot{r}\,\dot{\theta}$; $\mathbf{i} = \cos\theta\,\hat{\mathbf{e}}_r - \sin\theta\,\hat{\mathbf{e}}_\theta$, $\mathbf{j} = \sin\theta\,\hat{\mathbf{e}}_r + \cos\theta\,\hat{\mathbf{e}}_\theta$.



(a) $v_r = \dot{r} = 150$ m/s $\quad v_\theta = r\dot{\theta} = -197.6$ m/s

Speed $v = \sqrt{v_r^2 + v_\theta^2} = 248.1$ m/s

(b) $a_r = 4.10 - 3800(-0.052)^2 = -6.175$ m/s^2

$a_\theta = 3800(-0.011) + 2(150)(-0.052) = -57.4$ m/s^2

$a = \sqrt{a_r^2 + a_\theta^2} = 57.73$ m/s^2

1. An airplane is tracked by radar. When the airplane is at P, the following radar measurements are recorded: $r = 3800$ m, $\dot{r} = 150$ m/s, $\ddot{r} = 4.10$ m/s^2, $\theta = 30^o$C, $\dot{\theta} =$ - 0.052 rad/s, $\ddot{\theta} =$ - 0.011 rad/s^2. Determine (a) the magnitude of the plane's speed v, (b) the magnitude of the plane's acceleration a, and (c) the angle of dive α of the plane at that position. Hint: $v_r = \dot{r}$, $v_\theta = r\,\dot{\theta}$, $a_r = \ddot{r} - r\,\dot{\theta}^2$, $a_\theta = r\,\ddot{\theta} + 2\,\dot{r}\,\dot{\theta}$; $\mathbf{i} = \cos\theta\,\hat{\mathbf{e}}_r - \sin\theta\,\hat{\mathbf{e}}_\theta$, $\mathbf{j} = \sin\theta\,\hat{\mathbf{e}}_r + \cos\theta\,\hat{\mathbf{e}}_\theta$.

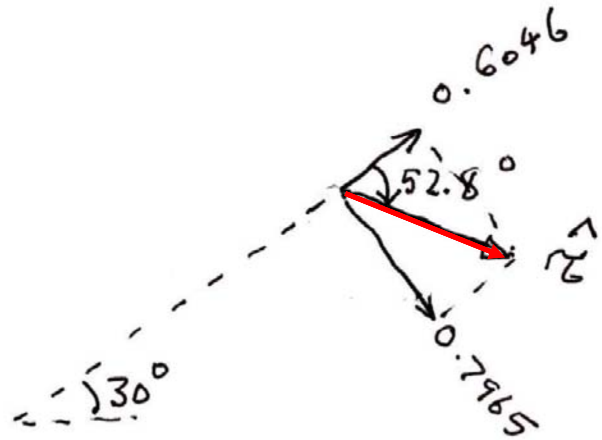
c) $v = 150\, e_r - 197.6\, e_\theta$

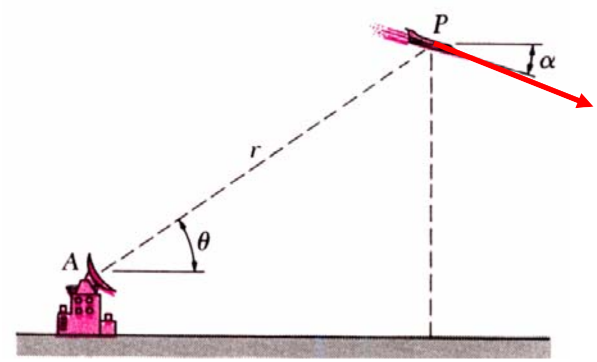
$\hat{\tau} = \dfrac{\vec{v}}{v} = 0.6046\,\hat{e}_r - 0.7965\,\hat{e}_\theta$

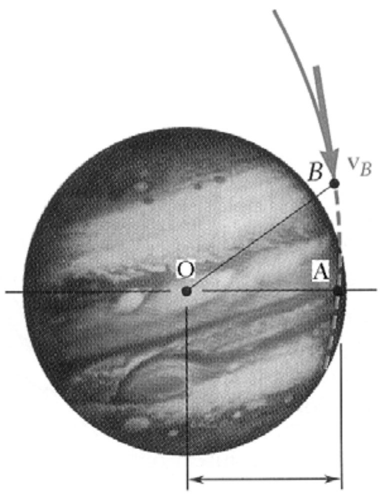
$\hat{i} = \cos 30^\circ\,\hat{e}_r - \sin 30^\circ\,\hat{e}_\theta$

$\cos\alpha = \hat{\tau}\cdot\hat{i} = 0.6046\cos 30^\circ + 0.7965 \sin 30^\circ = 0.9218$

$\alpha = 22.8^\circ$





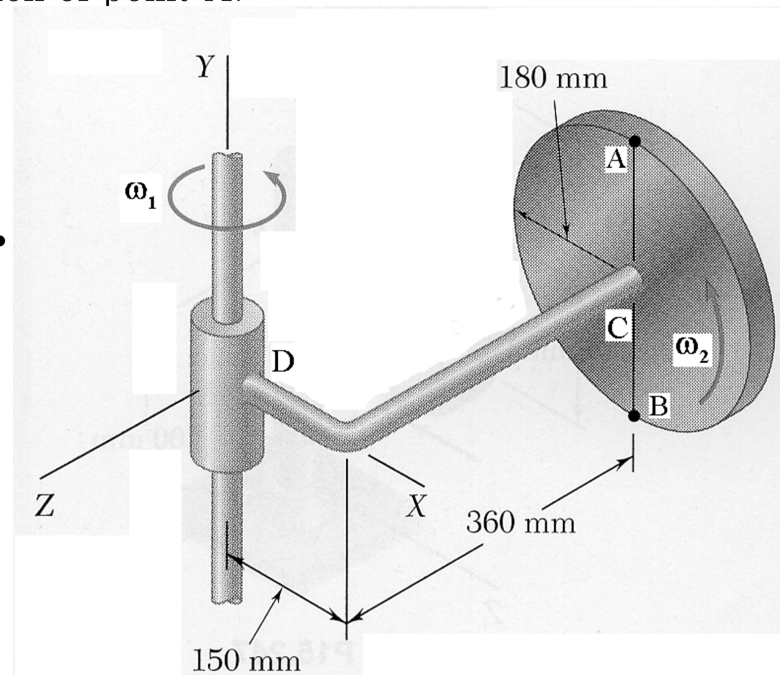


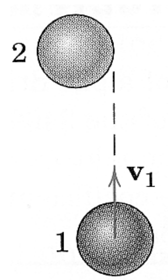
2. As a spacecraft approaches the planet Jupiter, it releases a probe which is to enter the planet's atmosphere at point B at an altitude of 450 km above the surface of the planet. The trajectory of the probe is a hyperbola of eccentricity $\epsilon = 1.031$. Knowing that the radius and the mass of Jupiter are 71.492×10^3 km and 1.9×10^{27} kg, respectively, and that the velocity v_B of the probe at B forms an angle of 82.9^o with the direction of OA, determine (a) the angle AOB, (b) the speed v_B of the probe at B. The perigee, A, is 70.8×10^{3} km from the center of Jupiter. Constant of gravitation = G = 66.73×10^{-12} $m^3/kg.s^2$.

3. A disk of 180 mm radius rotates at the constant rate $\omega_2 = 12$ rad/s with respect to arm CD, which itself rotates at the constant rate $\omega_1 = 8$ rad/s about the Y axis. Determine at the position shown, (a) the angular velocity of the disk, (b) the angular acceleration of the disk, (c) the velocity of point A on the rim of the disk, and (d) the acceleration of point A.

$$\text{velocity} = \frac{dR}{dt} + \omega \times \mathbf{r} + \dot{\mathbf{r}}$$

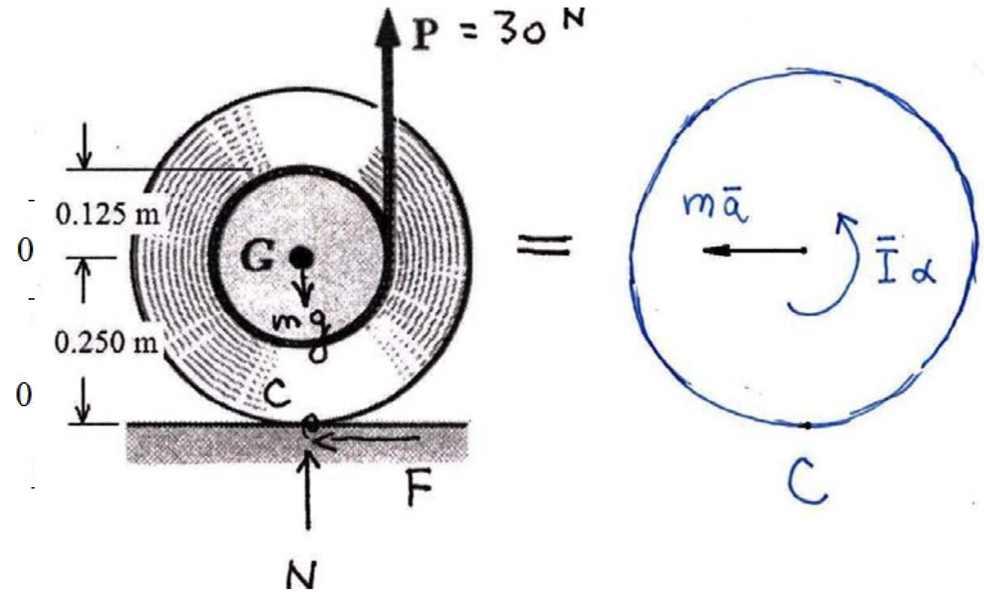
$$\text{accelaration} = \frac{d^2R}{dt^2} + \dot{\omega} \times \mathbf{r} + \omega \times (\omega \times \mathbf{r}) + \mathbf{2}\,\omega \times \dot{\mathbf{r}} + \ddot{\mathbf{r}}$$





4. Ball 1 has a velocity $v_1 = 6$ m/s in the direction shown and collides with ball 2 of equal mass and diameter. Ball 2 is initially at rest. If the coefficient of restitution is $e = 0.6$, determine (a) the speed and its direction of ball 1 after the impact, (b) the speed and its direction of ball 2 after the impact, and (c) the fraction of energy loss due to the impact.

5. A cord is wrapped around the inner drum and pulled vertically with a force $P = 30$ N. The wheel has a total mass 7 kg and a radius of gyration of $r_g = 0.19$ m. The radii of the drum and the wheel are 0.125 m and 0.250 m, respectively. Knowing that the wheel rolls without sliding, determine (a) the angular acceleration of the wheel, and (b) the minimum value of the coefficient of static friction, μ_s, compatible with this motion. Hint: $\overline{I} = mr_g^2$



$$\overline{I} = 7 \times (0.19)^2$$
$$= 0.2527 \text{ Kg-m}^2$$

<u>Kinetics</u>

$\leftarrow$: $F = m\bar{a}$

C ↺: $30(0.125) = \overline{I}\alpha + 0.25\, m\bar{a}$

<u>Kinematics</u>

$\bar{a} = 0.25\,\alpha$

Kinetics

$\leftarrow$: $F = m\bar{a}$

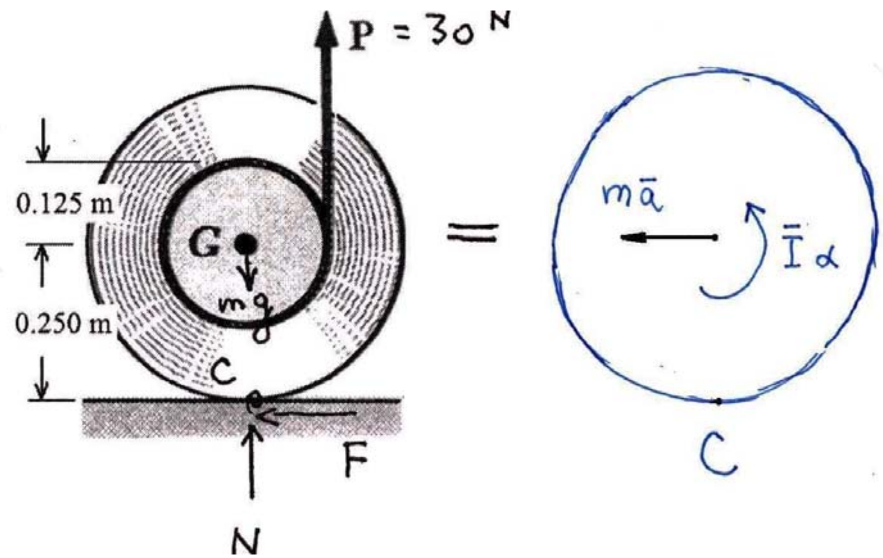
C ↺: $30(0.125) = \bar{I}\alpha + 0.25\, m\bar{a}$

Kinematics

$\bar{a} = 0.25\,\alpha$

(a) $3.75 = (0.2527 + 0.25 \times 7 \times 0.25)\alpha$

$$\alpha = \frac{3.75}{0.6902} = 5.433 \text{ rad/s}$$

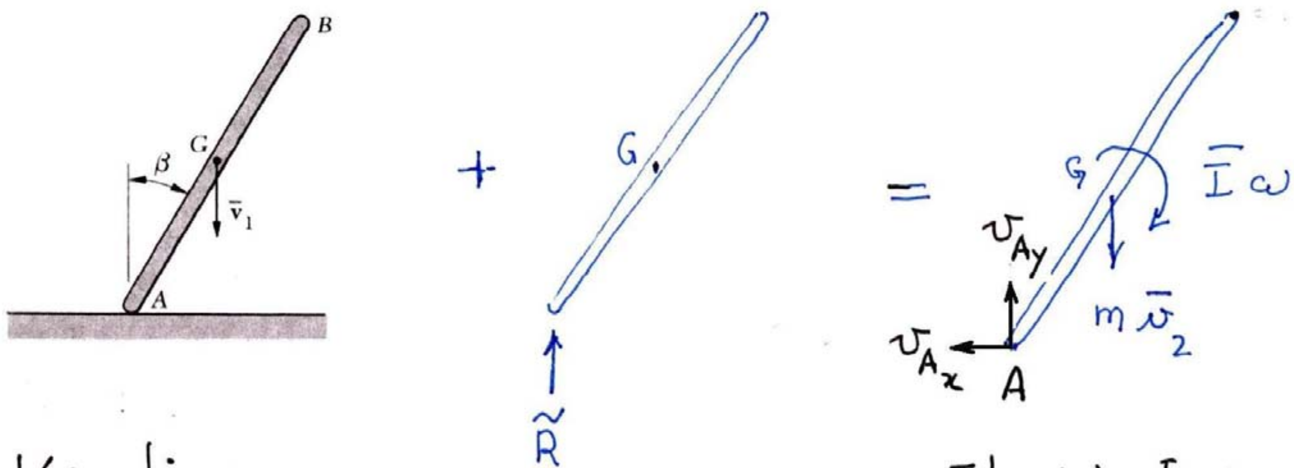


(b) $F = 7 \times 0.25\,\alpha = 9.508 \text{ N}$

$\uparrow$: $P + N - mg = 0$, $N = 7 \times 9.81 - 30 = 38.67 \text{ N}$

$$\mu_s = \frac{F}{N} = \frac{9.508}{38.67} = 0.2459$$

6. The slender rod AB of length $L = 6$ m forms an angle $\beta = 30^o$ with the vertical as it strikes the frictionless surface shown with a vertical velocity $\overline{v}_1 = 2$ m/s and no angular velocity. Assuming that the impact is perfectly elastic, find (a) the angular velocity of the rod immediately after the impact, and (b) the velocity of B immediately after the impact. Hint: $\overline{I} = \frac{1}{12}mL^2$



Kinetics

$\downarrow$: $m\overline{v}_1 - \tilde{R} = m\overline{v}_2$ ----(1)

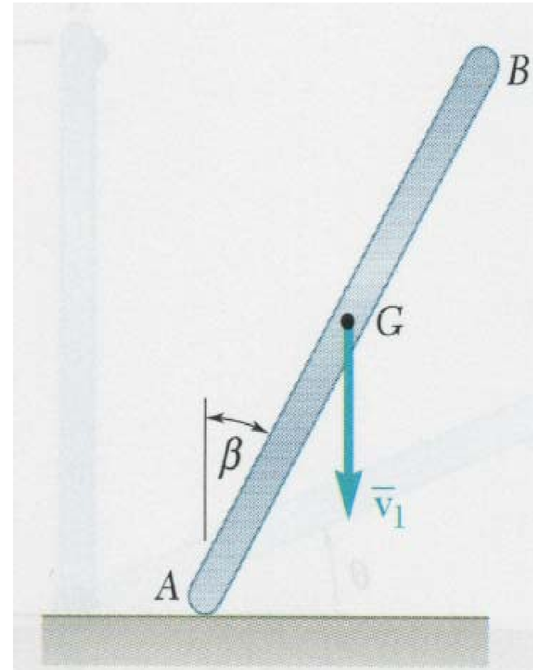
G ↻: $0 + \left(\frac{L}{2}\sin\beta\right)\tilde{R} = \overline{I}\omega$ ----(2)

Elastic Impact at A

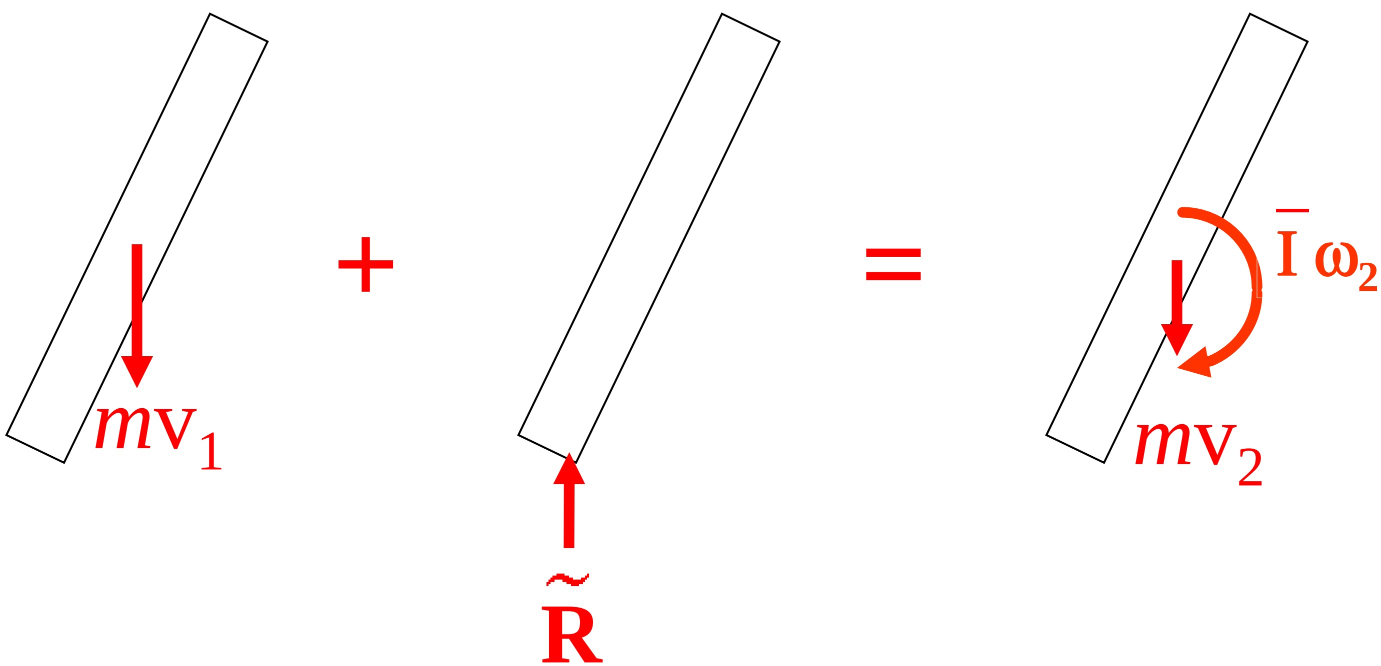
$v_{Ay} = \overline{v}_1$

Kinematics $\vec{v}_A = \vec{v}_G + \vec{\omega} \times \vec{r}_{AG}$

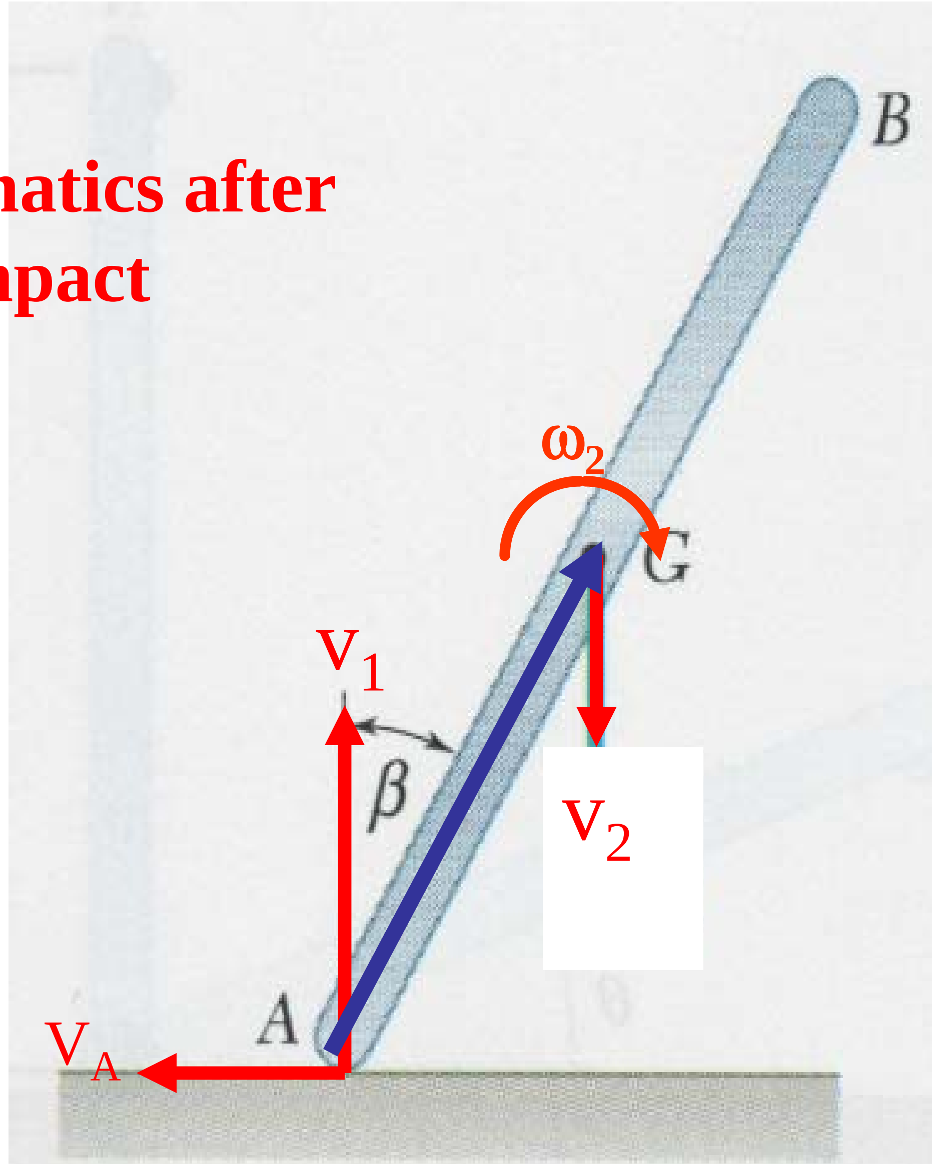
17.109 The slender rod *AB* of length *L* forms an angle β with the vertical as it strikes the frictionless surface shown with a vertical velocity $\bar{\mathbf{v}}_1$ and no angular velocity. Assuming that the impact is perfectly elastic, derive an expression for the angular velocity of the rod immediately after the im-



Impact Diagram

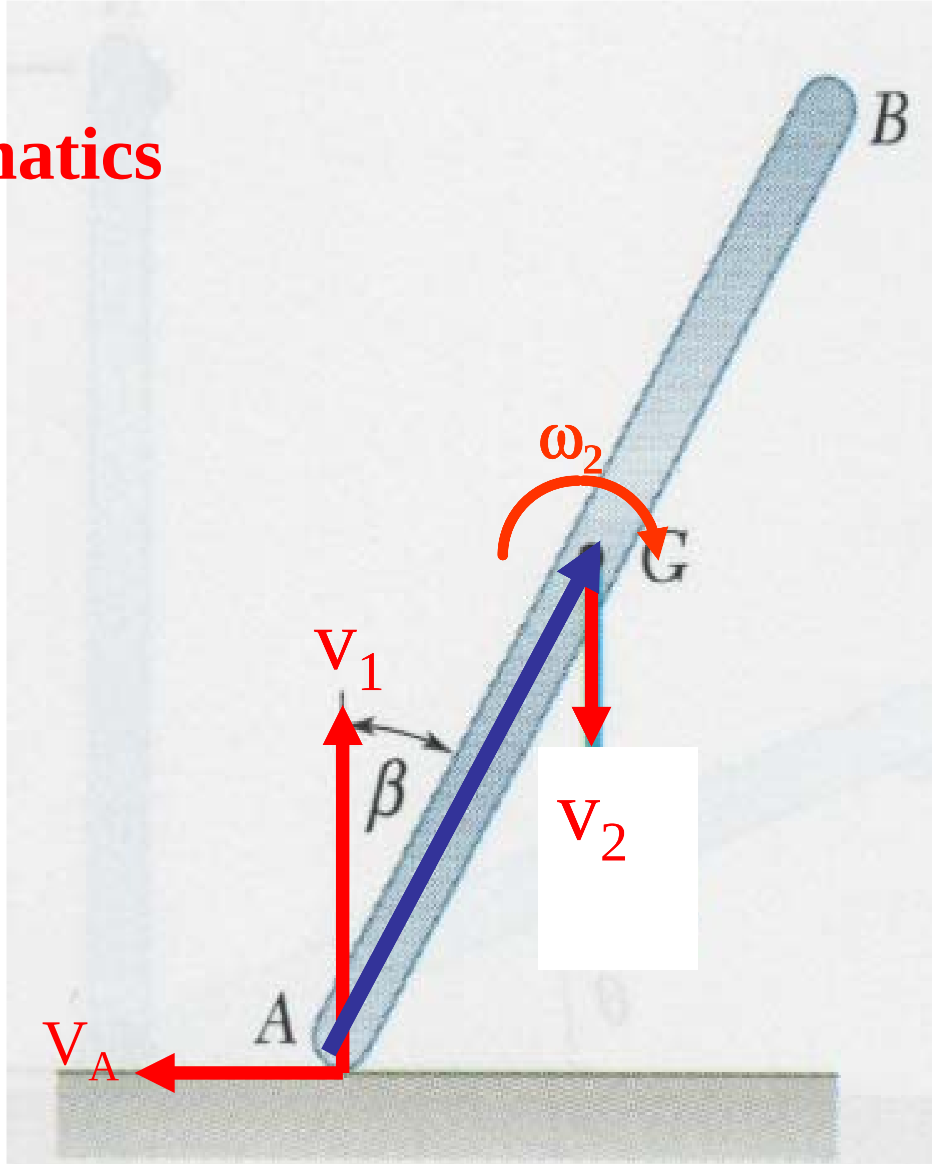


Kinematics after the Impact



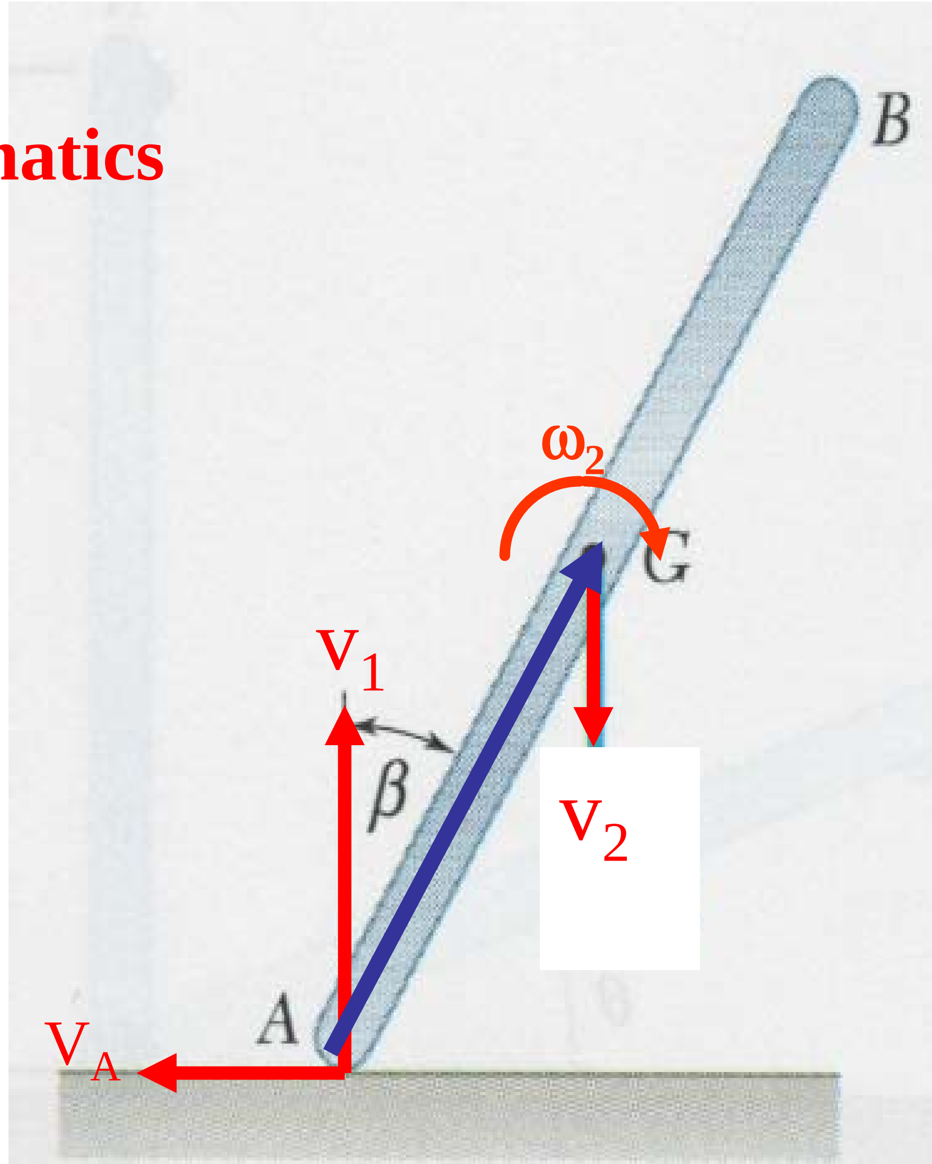
$$\mathbf{v}_G = v_A\,\mathbf{i} + v_1\,\mathbf{j} + \boldsymbol{\omega}_2 \times \mathbf{r}_{G/A}$$

Kinematics



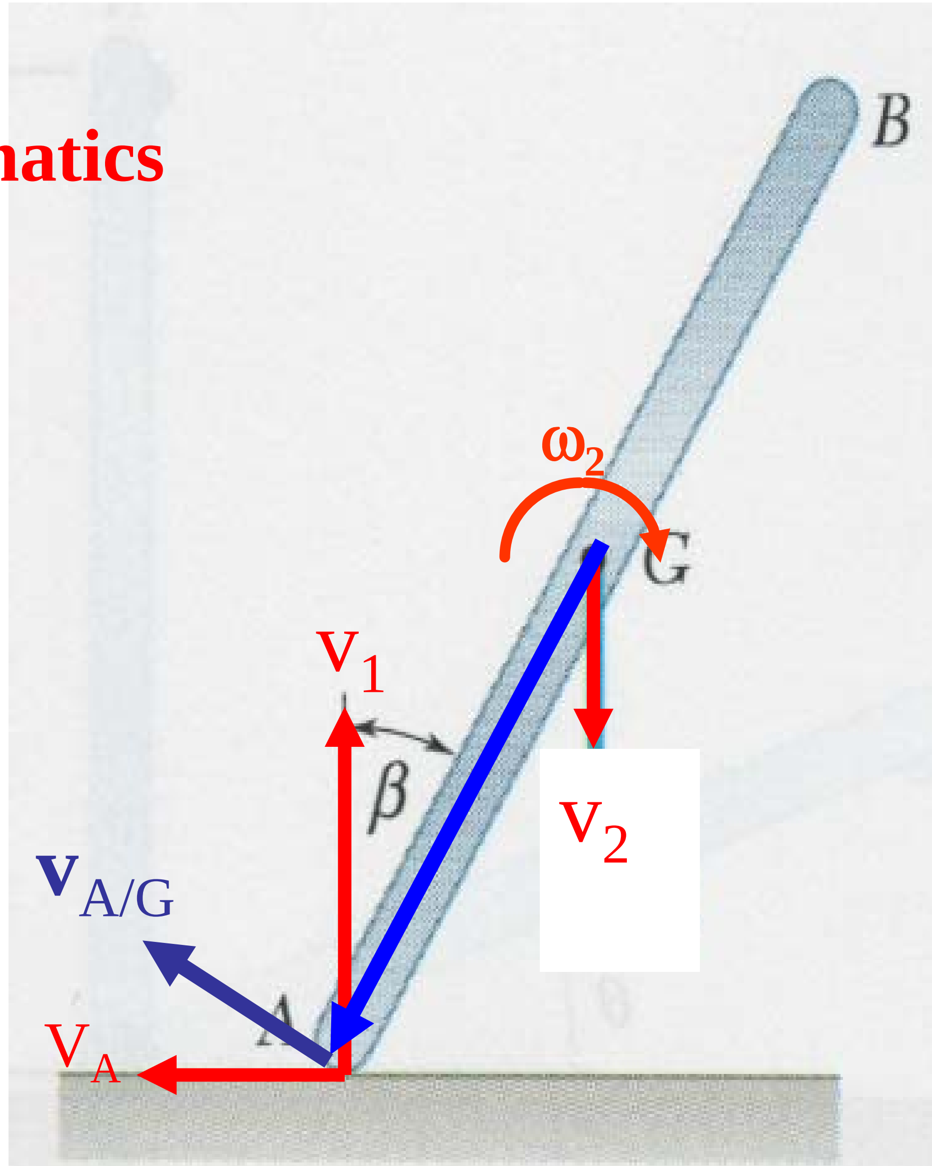
$$-v_2\,\mathbf{j} = v_A\,\mathbf{i} + v_1\,\mathbf{j} - \omega_2\mathbf{k} \times r_{G/A}(\sin\beta\,\mathbf{i} + \cos\beta\,\mathbf{j})$$

Kinematics



$$- v_2 = + v_1 - \omega_2 \, r_{G/A} \sin\beta$$

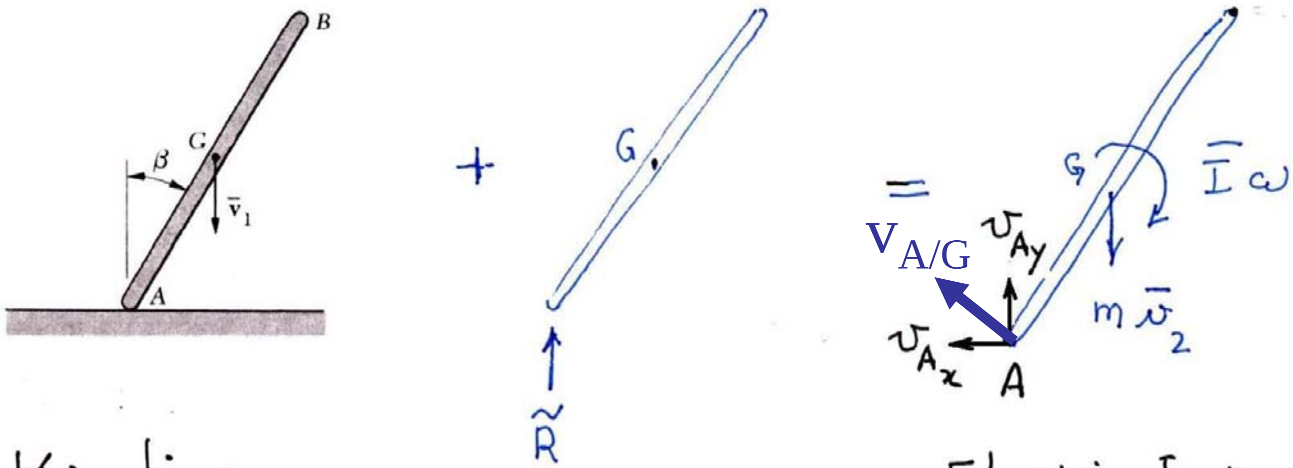
Kinematics



$$\mathbf{v}_A = \mathbf{v}_G + \mathbf{v}_{A/G}$$

$$|\mathbf{v}_{A/G}| = \tfrac{1}{2} L\omega_2$$

$$v_1 = - v_2 + (\tfrac{1}{2}L)\,\omega_2 \sin\beta$$



$|v_{A/G}| = \frac{1}{2}L\omega$

Kinetics

$\downarrow$: $m\bar{v}_1 - \tilde{R} = m\bar{v}_2$ ----(1)

G ↻: $0 + \left(\frac{L}{2}\sin\beta\right)\tilde{R} = \bar{I}\omega$ ----(2)

Elastic Impact at A

$v_{Ay} = \bar{v}_1$

Kinematics

$\vec{v}_A = \vec{v}_G + \vec{\omega} \times \vec{r}_{AG}$

y-component: $\bar{v}_1 = -\bar{v}_2 + \frac{L}{2}\omega\sin\beta$ ---(3)

Eq.(2) & (1):

$\left(\frac{L}{2}\sin\beta\right)m\left(\bar{v}_1 - \bar{v}_2\right) = \frac{1}{12}mL^2\omega$ ---(4)

$\bar{v}_1 + \bar{v}_2 = \frac{L}{2}\omega\sin\beta$

$\bar{v}_1 - \bar{v}_2 = \frac{L}{6}\omega\frac{1}{\sin\beta}$

$2\bar{v}_1 = \frac{L}{6}\omega\left(3\sin\beta + \frac{1}{\sin\beta}\right)$

$\bar{v}_2 = \frac{2}{7}\hat{j}$

(a) $\omega = \frac{4}{3\times0.5 + \frac{1}{0.5}} = \frac{4}{1.5+2} = \frac{8}{7} = 1.143$ rad/s

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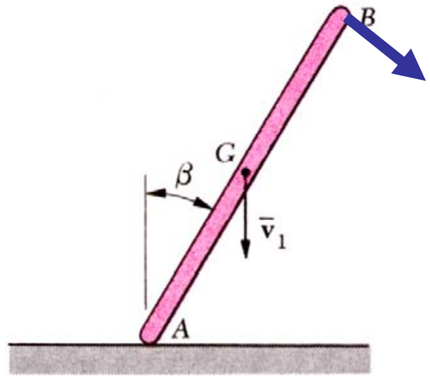

$|v_{B/G}| = ½L\omega$

(b) $\vec{v}_B = \vec{v}_G + \vec{v}_{B/G} = -\bar{v}_2 \hat{j} - \frac{L}{2}\omega \sin\beta \hat{j} + \frac{L}{2}\omega \cos\beta \hat{i}$

$= 0.2857 \hat{j} - 1.715 \hat{j} + 2.970 \hat{i} = 2.970 \hat{i} - 1.429 \hat{j}$