LAST NAME Solution

FIRST NAME ____________________

STUDENT NO. ____________________

Department of Civil Engineering and Applied Mechanics
McGill University

ANALYTICAL MECHANICS, CIVE281

Test No.1

Examiners: Prof. V. H. Chu
Prof. S. Babarutsi

Date: Wednesday, October 5, 2005
Time: 8:30 a.m. - 9:25 a.m.

Answer on the space provided below the question. Continue on the facing page if more space is needed.

QUESTION	MARK
1 (50%)	
2 (50%)	
TOTAL	

1. (50%) At a general time t, a particle has a position vector $\mathbf{r} = 2\,\hat{\mathbf{i}} + \frac{1}{2}t^2\,\hat{\mathbf{j}} + t\,\hat{\mathbf{k}}$, in which $\mathbf{r}$ in m and t in s. Find at time $t = 1$ s, (i) the velocity and acceleration vectors, (ii) the unit tangent vector to path $\hat{\tau}$, (iii) the tangential component of acceleration a_t, (iv) the normal component of acceleration a_n, (v) the radius of curvature of path ρ, and (vi) the unit normal vector $\hat{\mathbf{n}}$

$$\vec{r} = 2\hat{i} + \frac{1}{2}t^2\hat{j} + t\hat{k}$$

(i) 8%

$$\vec{v} = t\hat{j} + \hat{k}, \quad v = \sqrt{1^2 + 1^2} = \sqrt{2}\ \text{m/s}$$

$$\vec{a} = \hat{j}, \quad a = 1\ \text{m/s}^2$$

8 (ii)

$$\hat{\tau} = \frac{\vec{v}}{v} = \frac{1}{\sqrt{2}}\hat{j} + \frac{1}{\sqrt{2}}\hat{k}$$

8 (iii)

$$a_\tau = \vec{a}\cdot\hat{\tau} = \frac{1}{\sqrt{2}}\ \text{m/s}^2$$

(iv) 8

$$a_n = \sqrt{a^2 - a_\tau^2} = \sqrt{1 - \frac{1}{2}} = \frac{1}{\sqrt{2}}\ \text{m/s}^2$$

8 (v)

$$\rho = \frac{v^2}{a_n} = \frac{2}{\frac{1}{\sqrt{2}}} = 2\sqrt{2}\ \text{m}$$

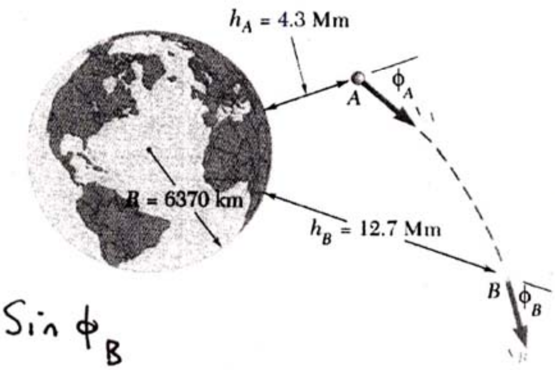
10 (vi)

$$\vec{a} = a_\tau\hat{\tau} + a_n\hat{n}$$

$$\hat{n} = \frac{\vec{a} - a_\tau\hat{\tau}}{a_n} = \frac{\hat{j} - \frac{1}{2}\hat{j} - \frac{1}{2}\hat{k}}{\frac{1}{\sqrt{2}}}$$

$$= \frac{\sqrt{2}}{2}\hat{j} - \frac{\sqrt{2}}{2}\hat{k}$$

2. (50%) At an altitude $h_A = 4.30 \times 10^6$ m above the earth surface, the velocity of the space probe fired from earth has a magnitude of $v = 32.5 \times 10^6$ m/h and an angle of $\phi_A = 60^o$ with the radial direction. Find (a) the angular momentum per unit mass of the space probe, (b) the θ-component of the velocity, v_θ, as the probe passes through the point B at the altitude $h_B = 12.7 \times 10^6$ m above the earth surface. The earth radius is $R = 6.37 \times 10^6$ m



$r_A = R + h_A = 10.67 \times 10^6$ m

$r_B = R + h_B = 19.07 \times 10^6$ m

25%

(a) $h = r_A v_A \sin\phi_A = r_B v_B \sin\phi_B$

$= 10.67 \times 10^6 \times 32.5 \times 10^6 \sin 60^\circ$

$= 3.003 \times 10^{14}\ m^2/h = 8.342 \times 10^{10}\ m^2/s$

25%

(b) $(v_B)_\theta = \dfrac{h}{r_B} = 1.575 \times 10^7\ m/h = 4.374 \times 10^3\ m/s$