Time Value Conepts & Applications

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Chapter Outline

Introduction to Valuation: The Time Value of Money

2 Future Value for a Lump Sum

- What is the future value of $100 in 3 years if r = 10%?
- Notice that
  - 1. $110 = $100 × (1 + .10)
  - 2. $121 = $110 × (1 + .10) = $100 × 1.1 × 1.1 = $100 × 1.1^2
  - 3. $133.10 = $121 × (1 + .10) = $100 × 1.1 × 1.1 × 1.1
    = $100 × $(1 + .10)^3$
- In general, the future value, FV_t, of $1 invested today at r% for t periods is

$$FV_t = \$1 \times (1 + r)^t$$

- The expression $(1 + r)^t$ is the ***future value interest factor.***

3 Quick Quiz

- Want to be a millionaire?
- No problem!
- Suppose you are currently 21 years old, and can earn 10 percent on your money

  (about what the typical common stock has averaged over the last six decades - but more on that later).
- How much must you invest *today* in order to accumulate $1 million by the time you reach age 65?

4 Sol Quick Quiz

- Once again, we first define the variables:

  FV = $1 million   r = 10 percent

  t = 65 - 21 = 44 years   PV = ?
- Set this up as a future value equation and solve for the present value:

  $1 million = PV x $(1.10)^{44}$

  PV = $1 million/$(1.10)^{44}$ = $15,091.
- Of course, we've ignored taxes and other complications, but stay tuned - right now you need to figure out where to get $15,000!

5 Present Value for a Lump Sum

- Q. Suppose you need $20,000 in three years to pay your college tuition. If you can earn 8% on your money, how much do you need today?
- A. Here we know the future value is $20,000, the rate (8%), and the number of periods (3). What is the unknown present amount (i.e., the ***present value***)? From before:

  FV_t = PV x $(1 + r)^t$
  $20,000 = PV x $(1.08)^3$

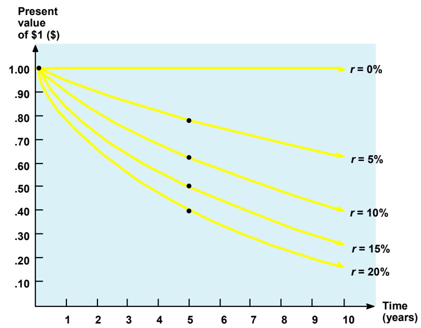
  Rearranging:

  PV = $20,000/$(1.08)^3$
  = $15,876.64

The PV of a $1 to be received in *t* periods when the rate is *r* is

$$PV = \$1/(1 + r)^t$$

6 Present Value of $1 for Different Periods and Rates



7 Quick Quiz

- Suppose you deposit $5000 today in an account paying *r* percent per year. If you will get $10,000 in 10 years, what *rate of return* are you being offered?
- Set this up as present value equation:

  FV = $10,000 PV = $ 5,000 t = 10 years

  $PV = FV_t/(1 + r)^t$

  $\$5000 = \$10{,}000/(1 + r)^{10}$

- Now solve for *r*:

  $(1 + r)^{10} = \$10{,}000/\$5{,}000 = 2.00$

  $r = (2.00)^{1/10} - 1 = .0718$ = 7.18 percent

7.1 The Rule of 72

- The "Rule of 72" is a handy rule of thumb that states the following:

  If you earn *r* % per year, your money will double in about 72/*r* % years.

  So, for example, if you invest at 6%, your money will double in 12 years.

- Why do we say "about?" Because at higher-than-normal rates, the rule breaks down.

  What if r = 72%? ⇒ FVIF(72,1) = 1.72, not 1.00

  And if r = 36%? ⇒ FVIF(36,2) = 1.8496,not 2.00

  The lesson? The Rule of 72 is a useful rule of thumb, but it is *only* a rule of thumb!

8 Summary of Time Value Calculations

I. Symbols:

PV = Present value, what future cash flows are worth today

FV_t = Future value, what cash flows are worth in the future

r = Interest rate, rate of return, or discount rate per period

t = number of periods

C = cash amount

II. Future value of C dollars invested at r percent per period for t periods:

$FV_t = C \times (1 + r)^t$

The term $(1 + r)^t$ is called the future value interest factor and often abbreviated $FVIF_{r,t}$ or FVIF(r,t).

9 Summary of Time Value Calculations (concluded)

III. Present value of C dollars to be received in t periods at r percent per period:

$PV = C/(1 + r)^t$

The term $1/(1 + r)^t$ is called the *present value interest factor* and is often abbreviated $PVIF_{r,t}$ or PVIF(r,t).

IV. The basic present equation giving the relationship between present and future value is:

$PV = FV_t/(1 + r)^t$

10 Quick Quiz

- Now let's see what we remember!

1. Which of the following statements is/are true?
   - Given r and t greater than zero, future value interest factors ($FVIF_{r,t}$) are always greater than 1.00.
   - Given r and t greater than zero, present value interest factors ($PVIF_{r,t}$) are always less than 1.00.
   - Given r and t greater than zero, annuity present value interest factors ($PVIFA_{r,t}$) are always less than t.
2. True or False: For given levels of r and t, $PVIF_{r,t}$ is the reciprocal of $FVIF_{r,t}$.
3. All else equal, the higher the discount rate, the (lower/higher) the present value of a set of cash flows.

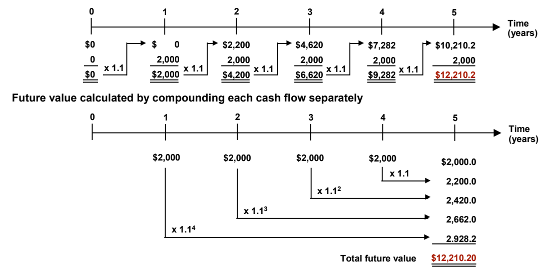
11 Quick Quiz Sol

1. All three statements are true.

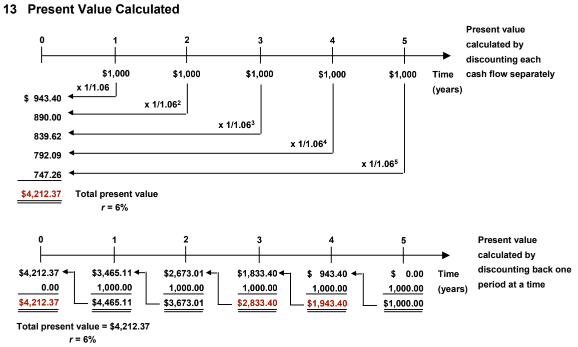
2. This statement is also true. $PVIF_{r,t}$ = $1/FVIF_{r,t}$.

3. The answer is *lower* - discounting cash flows at higher rates results in lower present values. And compounding cash flows at higher rates results in higher future values.

12 Future Value Calculated



13 Present Value Calculated



14 Annuities and Perpetuities -- Basic Formulas

- Annuity Present Value

  PV = C x {1 - [1/(1 + r)^t]}/r

- Annuity Future Value

  FV_t = C x {[(1 + r)^t - 1]/r}

- Perpetuity Present Value

  PV = C/r

- The formulas above are the basis of many of the calculations in Finance. It will be worthwhile to keep them handy!

15 - Examples: Annuity Present Value

Example: Finding C

- Q. You want to buy a Mazda Miata to go cruising. It costs $17,000. With a 10% down payment, the bank will loan you the rest at 12% per year (1% per month) for 60 months. What will your payment be?
- A. You will borrow ______× $17,000 = $______ . This is the amount today, so it's the_______ . The rate is______ , and there are _______ periods:

$______ = C× {1 - [1/(1 + r)^t]}/ .01
= C× {1 - .55045}/.01
= C× 44.955

C = $15,300/44.955
C = $340.34____________

15 - Examples: Annuity Present Value

Example: Finding C

- Q. You want to buy a Mazda Miata to go cruising. It costs $17,000. With a 10% down payment, the bank will loan you the rest at 12% per year (1% per month) for 60 months. What will your payment be?

- A. You will borrow .9 X $17,000 = $15,300. This is the amount today . The rate is 1% , and there are 60 periods:

$_15,300_ = C× {1 - [1/(1 + r)^t]}/.01
= C× {1 - .55045}/.01
= C× 44.955

C = $15,300/44.955
C = $340.34____________

16 - Examples: Annuity Present Value

Example: Finding t

- Q. Suppose you owe $2000 on a VISA card, and the interest rate is 2% per month. If you make the minimum monthly payments of $50, how long will it take you to pay it off?
- A. A *long* time:

  $2000 = $50 × {____________)}/.02

  t = 81.274 *months* or about 6.77 years

17 Quick Quiz

Annuity Present Value

- Suppose you need $20,000 each year for the next three years to make your tuition payments.

  Assume you need the first $20,000 in exactly one year. Suppose you can place your money in a savings account yielding 8% compounded annually. How much do you need to have in the account *today*?

  (Note: Ignore taxes, and keep in mind that you don't want any funds to be left in the account after the third withdrawal, nor do you want to run short of money.)

18 Solution to Quick Quiz

Annuity Present Value - Solution

Here we know the periodic cash flows are $20,000 each. Using the most basic approach:

PV = $20,000/1.08 + $20,000/1.08^2 + $20,000/1.08^3
= $18,518.52 + $17146.78 + $15,876.65
= $51,541.94

Here's a shortcut method for solving the problem using the *annuity present value factor*:

PV = $20,000 × {____________}/__________
= $20,000 × 2.577097
= $_______________

19 Example: Annuity Future Value

- Previously we determined that a 21-year old could accumulate $1 million by age 65 by investing $15,091 today and letting it earn interest (at 10%compounded annually) for 44 years.

  Now, rather than plunking down $15,091 in one chunk, suppose she would rather invest smaller amounts annually to accumulate the million. If the first deposit is made in one year, and deposits will continue through age 65, how large must they be?
- Set this up as a FV problem:

  $1,000,000 = C× [(1.10)^{44} - 1]/.10

  C = $1,000,000/652.6408 = $1,532.24

  Becoming a millionaire just got easier!

20 Example: Perpetuity Calculations

- A *perpetuity* is an annuity that never ends.
- Suppose we expect to receive $1000 per year *forever*. This is called a *perpetuity*.
- In this case, the PV is easy to calculate, given r=6%:

  PV = C/r = $1000/____ = $16,666.66…

20 Summary of Annuity and Perpetuity Calculations

I. Symbols

PV	=	Present value, what future cash flows bring today
FV_t	=	Future value, what cash flows are worth in the future
r	=	Interest rate, rate of return, or discount rate per period
t	=	Number of time periods
C	=	Cash amount

II. FV of C per period for t periods at r percent per period:

$FV_t = C \times \{[(1 + r)^t - 1]/r\}$

III. PV of C per period for t periods at r percent per period:

$PV = C \times \{1 - [1/(1 + r)^t]\}/r$

IV. PV of a perpetuity of C per period:

$PV = C/r$

21 Compounding Periods, EARs, and APRs

Effective Interest Rate (EAR) = $[1 + r/m]^m - 1$

Compounding period (t)	Number of times compounded(m)	Effective annual rate
Year	1	10.00000%
Quarter	4	10.38129
Month	12	10.47131
Week	52	10.50648
Day	365	10.51558
Hour	8,760	10.51703
Minute	525,600	10.51709

22 Compounding Periods, EARs, and APRs (continued)

- **EARs and APRs**
- **Q.** If a rate is quoted at 16%, compounded semiannually, then the actual rate is 8% per six months. Is 8% per six months the same as 16% per year?
- **A.** If you invest $1000 for one year at 16%, then you'll have $1160 at the end of the year. If you invest at 8% per period for two periods, you'll have
- FV = $1000 × (1.08)^2$
  = $1000 × 1.1664
  = $1166.40,

  or $6.40 more. Why? What rate per year is the same as 8% per six months?

23 Compounding Periods, EARs, and APRs (concluded)

- The *Effective Annual Rate* (EAR) is 16.64%. The "16% compounded semiannually" is the quoted or stated rate, not the effective rate.
- By law, in consumer lending, the rate that must be quoted on a loan agreement is equal to the rate per period multiplied by the number of periods. This rate is called the Annual percentage rate (APR)
- **Q.** A bank charges 1% per month on car loans. What is the APR? What is the EAR?
- **A.** The APR is 1 x 12 = 12%. The EAR is:

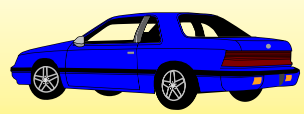
  The EAR = 1.126825 - 1 = 12.6825%

  The APR is thus a quoted rate, not an effective rate!

24 Example: Cheap Financing or Rebate?

SALE! SALE!

5%* FINANCING OR $500 REBATE
FULLY LOADED MUSTANG



only $10,999

*5% APR on 36 month loan.

Banks are making 10% car loans, should you choose the 5% financing or $500 rebate?

25 Example: Cheap Financing or Rebate?

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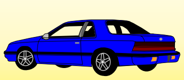

only $10,999

*5% APR on 36 month loan.

TF Banks are making 10% car loans, should you choose the 5% financing or $500 rebate?

- Assuming no down payment and a 36 month loan:
  - Bank: PV = $10,999 - 500 = $10,499, r = 10/12, t = 36

    $$\$10{,}499 = C \times \left\{ \frac{1 - PVIF(.10/12, 36)}{.10/12} \right\}$$

    C = 338.77
  - 5% APR: PV = $10,999, r = .05/12, t = 36

    $$\$10{,}999 = C \times \left\{ \frac{1 - PVIF(.05/12, 36)}{.05/12} \right\}$$

    $$C = \frac{\$10{,}999}{33.3657} = 329.65$$
  - The best deal? Take the 5% APR!

26 Quick Quiz

How to lie, cheat, and steal with interest rates:

RIPOV RETAILING
Going out *for* business sale!

$1000 instant credit!
12% simple interest!
Three years to pay!
Low, low monthly payments!

Assume you buy $1,000 worth of furniture from this store and agree to the above credit terms. What is the APR of this loan? The EAR?

 Solution to Quick Quiz

- Your payment is calculated as:
  - 1. Borrow $1000 today at 12% per year for three years, you will owe $1000 + $1000(.12)(3) = $1360.
  - 2. To make it easy on you, make 36 low, low payments of $1360/36 = $37.78.
  - 3. Is this a 12% loan?_______

$1,000 = $37.78 x (1 - 1/(1 + r)^{36})/r

r = 1.766% *per month*

APR = 12(1.766%) = 21.19%

EAR = 1.01766^{12} - 1 = 23.38% (!)

 Example: Amortization Schedule - **Fixed Principal**

Loan = $5,000 rate = 9% term = 5 years

Year	Beginning Balance	Total Payment	Interest Paid	Principal Paid	Ending Balance
1	$5,000	$1,450	$450	$1,000	$4,000
2	4,000	1,360	360	1,000	3,000
3	3,000	1,270	270	1,000	2,000
4	2,000	1,180	180	1,000	1,000
5	1,000	1,090	90	1,000	0
Totals		$6,350	$1,350	$5,000	

 Example: Amortization Schedule - **Fixed Payments**

Loan = $5,000 rate = 9% term = 5 years

Year	Beginning Balance	Total Payment	Interest Paid	Principal Paid	Ending Balance
1	$5,000.00	$1,285.46	$ 450.00	$ 835.46	$4,164.54
2	4,164.54	1,285.46	374.81	910.65	3,253.88
3	3,253.88	1,285.46	292.85	992.61	2,261.27
4	2,261.27	1,285.46	203.51	1,081.95	1,179.32
5	1,179.32	1,285.46	106.14	1,179.32	0.00
Totals		$6,427.30	$1,427.31	$5,000.00	

 One last example.

- True story: An automobile dealer gave the following offer; Deposit $50,000 today and you will receive a brand new Mercedes Benz with a value of $25,000. In five years time you get your money $50,000 back and get to keep the car. It was 1980 and interest rates were 20%. What do you think of this deal?

PV of car = $25,000

PV of $50,000 at 20%, 5 years = $20094

PV of deposit = -$50,000

Total present value to ‘buyer’ = -$4906

Not a very good deal...