## Interest Rates and Bond Valuation

## Chapter Organization

- Bonds and Bond Valuation

■ More on Bond Features
■ Bond Ratings
■ Some Different Types of Bonds

T7.1 Chapter Outline (concluded)

Interest Rates and Bond Valuation

Chapter Organization

- Bond Markets
- Inflation and Interest Rates
- Summary and Conclusions
- Bond - evidence of debt issued by a corporation or a governmental body. A bond represents a loan made by investors to the issuer. In return for his/her money, the investor receives a legal claim on future cash flows of the borrower. The issuer promises to:
- Make regular coupon payments every period until the bond matures, and
- Pay the face/par/maturity value of the bond when it matures.
- Default - since the above-mentioned promises are contractual obligations, an issuer who fails to keep them is subject to legal action on behalf of the lenders (bondholders).

Bond Features (concluded)

- If a bond has five years to maturity, an $\$ 80$ annual coupon, and a $\$ 1000$ face value, its cash flows would look like this:

| Time | 0 | 1 | 2 | 3 | 4 | 5 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| Coupons |  | $\$ 80$ | $\$ 80$ | $\$ 80$ | $\$ 80$ | $\$ 80$ |
| Face Value |  |  |  |  | $\$$ | 1000 |
|  |  |  |  |  | $\$$ |  |
|  |  |  |  |  |  |  |

- How much is this bond worth? It depends on the level of current market interest rates. If the going rate on bonds like this one is $10 \%$, then this bond is worth $\$ 924.18$. Why? Stay tuned.

■ Suppose a bond currently sells for $\$ 932.90$. It pays an annual coupon of $\$ 70$, and it matures in 10 years. It has a face value of $\$ 1000$. What are its coupon rate, current yield, and yield to maturity (YTM)?

- 1. The coupon rate (or just "coupon") is the annual dollar coupon expressed as a percentage of the face value:
Coupon rate $=\$ 70 / \$$ $\qquad$ $=\ldots \quad \%$
- 2. The current yield is the annual coupon divided by the current market price of the bond:

Current yield = \$____=7.5\%
Under what conditions will the coupon rate and current yield be the same? Stay tuned.

## Bond Rates and Yields (concluded)

■ 3. The yield to maturity (or "YTM") is the rate that makes the price of the bond just equal to the present value of its future cash flows. It is the unknown $r$ in:

$$
\$ 932.90=\$ \ldots \quad \times\left[1-1 /(1+r)^{10}\right] / r+\$ \ldots \quad /(1+r)^{10}
$$

The only way to find the YTM is trial and error:
a. $\operatorname{Try} 10 \%: \$ 70 \times\left[\left(1-1 /(1.10)^{10}\right] / .10+\$ 1000 /(1.10)^{10}=\$ 816\right.$
b. $\operatorname{Try} 9 \%: \$ 70 \times\left[1-1 /(1.09)^{10}\right] / .09+\$ 1000 /(1.09)^{10}=\$ 872$
c. $\operatorname{Try} 8 \%: \$ 70 \times\left[1-1 /(1.08)^{10}\right] / .08+\$ 1000 /(1.08)^{10}=\$ 933$
( $\therefore$ ) The yield to maturity is $\mathbf{8 \%}$

## Valuing a Bond

- Assume you have the following information.

Barnhart, Inc. bonds have a $\$ 1,000$ face value
The promised annual coupon is $\$ 100$
The bonds mature in 20 years
The market's required return on similar bonds is $10 \%$

- 1. Calculate the present value of the face value

$$
=\$ 1,000 \times\left[1 / 1.10^{20}\right]=\$ 1,000 \times .14864=\$ 148.64
$$

- 2. Calculate the present value of the coupon payments

$$
=\$ 100 \times\left[1-\left(1 / 1.10^{20}\right)\right] / .10=\$ 100 \times 8.5136=\$ 851.36
$$

- 3. The value of each bond $=\$ 148.64+851.36=\$ 1,000$


## Example: A Discount Bond

- Assume you have the following information.

Barnhart, Inc. bonds have a $\$ 1,000$ face value
The promised annual coupon is $\$ 100$
The bonds mature in 20 years
The market's required return on similar bonds is $12 \%$

- 1. Calculate the present value of the face value

$$
=\$ 1,000 \times\left[1 / 1.12^{20}\right]=\$ 1,000 \times .10366=\$ 103.66
$$

- 2. Calculate the present value of the coupon payments

$$
=\$ 100 \times\left[1-\left(1 / 1.10^{20}\right)\right] / .10=\$ 100 \times 7.4694=\$ 746.94
$$

- 3. The value of each bond $=\$ 103.66+746.94=\$ 850.60$


## Example: A Premium Bond

- Assume you have the following information.

Barnhart, Inc. bonds have a $\$ 1,000$ face value
The promised annual coupon is $\$ 100$
The bonds mature in 20 years
The market's required return on similar bonds is $8 \%$

- 1. Calculate the present value of the face value

$$
=\$ 1,000 \times\left[1 / 1.08^{20}\right]=\$ 1,000 \times .21455=\$ 214.55
$$

- 2. Calculate the present value of the coupon payments

$$
=\$ 100 \times\left[1-\left(1 / 1.08^{20}\right)\right] / .08=\$ 100 \times 9.8181=\$ 981.81
$$

- 3. The value of each bond $=\mathbf{\$ 2 1 4 . 5 5} \boldsymbol{+} 981.81=\$ 1,196.36$
- Why do the bonds in this and the preceding example have prices that are different from par?


## Bond Price Sensitivity to YTM



The Bond Pricing Equation

- Bond Value = Present Value of the Coupons
+ Present Value of the Face Value
$=C \times\left[1-1 /(1+r)^{t}\right] / r+F \times 1 /(1+r)^{t}$
where: $\quad$ C = the promised coupon payment
F = the promised face value
$t=$ number of periods until the bond matures
$r=$ the market's required return, YTM


## Interest Rate Risk and Time to Maturity (Figure 7.2)



Value of a Bond with a 10\% Coupon Rate for Different Interest Rates and Maturities

## Bond Pricing Theorems

- The following statements about bond pricing are always true.
- 1. Bond prices and market interest rates move in opposite directions.
- 2. When a bond's coupon rate is (greater than / equal to / less than) the market's required return, the bond's market value will be (greater than / equal to / less than) its par value.

3. Given two bonds identical but for maturity, the price of the longer-term bond will change more than that of the shorter-term bond, for a given change in market interest rates.

- 4. Given two bonds identical but for coupon, the price of the lower-coupon bond will change more than that of the higher-coupon bond, for a given change in market interest rates.

Features of a May Department Stores Bond
Terms

| Amount of issue | $\$ 125$ million |
| :--- | :--- |
| Date of issue | $2 / 28 / 86$ |
| Maturity | $3 / 1 / 16$ |
| Annual coupon | 9.25 |

Offer price ..... 100

## Explanations

The company will issue $\$ 125$ million worth of bonds.

The bonds were sold on 2/28/86.
The principal will be paid in 30 years.
The denomination of the bonds is $\$ 1,000$. Each bondholder will receive $\$ 92.50$ per bond per year ( $9.25 \%$ of the face value).

The offer price will be $100 \%$ of the $\$ 1,000$ face value per bond.

Features of a May Department Stores Bond (concluded)

| Terms |  | Explanations |  |
| :--- | :--- | :--- | :--- |
| Coupon payment dates | $3 / 1,9 / 31$ |  | Coupons of $\$ 92.50 / 2=\$ 46.25$ will be <br> paid on these dates. |
| Security | None | The bonds are debentures. |  |
| Sinking fund | Annual, <br> beginning 3/1/97 | The firm will make annual payments <br> toward the sinking fund. |  |
| Call provision | Not callable <br> before $2 / 28 / 93$ | The bonds have a deferred call feature. |  |
| Call price | 106.48 initially, <br> declining to 100 | After 2/28/93, the company can buy <br> back the bonds for $\$ 1,064.80$ per bond, <br> declining to $\$ 1,000$ on 2/28/05. |  |
| Rating | Moody's A2 | This is one of Moody's higher ratings. <br> The bonds have a low probability <br> of default. |  |

The Bond Indenture

## The Bond Indenture

- The bond indenture is a three-party contract between the bond issuer, the bondholders, and the trustee. The trustee is hired by the issuer to protect the bondholders' interests. (What do you think would happen if an issuer refused to hire a trustee?)
- The indenture includes
- The basic terms of the bond issue
- The total amount of bonds issued
- A description of the security (if any)
- Repayment arrangements
- Call provisions
- Details of the protective covenants


## Bond Ratings

| Standard \& Poor's Moody's |  | Investment-Quality Bond Ratings |  |  |  | Low Quality, speculative, and/or "Junk" |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | High | rade |  | Grade | Low | ade | Very | ow G | Grade |
|  |  | AAA Aaa | AA $\mathrm{Aa}$ | A | $\begin{aligned} & \text { BBB } \\ & \text { Baa } \end{aligned}$ | $\begin{aligned} & \text { BB } \\ & \mathrm{Ba} \end{aligned}$ | $\begin{aligned} & \mathbf{B} \\ & \mathbf{B} \end{aligned}$ | $\begin{aligned} & \text { CCC } \\ & \text { Caa } \end{aligned}$ |  | $\begin{array}{ll} C & D \\ C & C \end{array}$ |
| Moody's | S\&P |  |  |  |  |  |  |  |  |  |
| Aaa | AAA | Debt rated Aaa and AAA has the highest rating. Capacity to pay interest and principal is extremely strong. |  |  |  |  |  |  |  |  |
| Aa | AA | Debt rated Aa and AA has a very strong capacity to pay interest and repay principal. Together with the highest rating, this group comprises the high-grade bond class. |  |  |  |  |  |  |  |  |
| A | A | Debt rated A has a strong capacity to pay interest and repay principal, although it is somewhat more susceptible to the adverse effects of changes in circumstances and economic conditions than debt in high rated categories. |  |  |  |  |  |  |  |  |

Bond Ratings (concluded)

| Baa | BBB | Debt rated Baa and BBB is regarded as having an <br> adequate capacity to pay interest and repay principal. <br> Whereas it normally exhibits adequate protection <br> parameters, adverse economic conditions or changing <br> circumstances are more likely to lead to a weakened <br> capacity to pay interest and repay principal for debt in <br> this category than in higher rated categories. These <br> bonds are medium-grade obligations. |
| :--- | :--- | :--- |
| Ba, B | BB, B C | Debt rated in these categories is regarded, on balance, as <br> predominantly speculative with respect to capacity to pay <br> interest and repay principal in accordance with the terms <br> of the obligation. BB and Ba indicate the lowest degree of <br> speculation, and CC and Ca the highest degree of <br> speculation. Although such debt will likely have some <br> quality and protective characteristics, these are out- <br> weighed by large uncertainties or major risk exposures <br> to adverse conditions. Some issues may be in default. |
| D | D | Debt rated D is in default, and payment of interest and/or <br> repayment of principal is in arrears |

## Sample Bond Quotations (Figure 7.3)

| Bonds <br> Supplied by RBC Dominion Securities Inc./ International from Reuters |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Coupon | Mat. Date | Bid S | YId\% |
| Government |  |  |  |  |
| Canada | 6.500 | Aug 01/99 | 101.36 | 5.23 |
| Canada | 7.750 | Sep 01/99 | 102.87 | 5.25 |
| Canada | 10.500 | Jul 01/00 | 109.96 | 5.27 |
| Canada | 7.250 | Jun 01/07 | 113.80 | 5.29 |
| OMHC | 8.200 | Jun 30/00 | 105.42 | 5.35 |
| OMHC | 5.100 | Jun 02/03 | 99.04 | 5.32 |
| Corporate |  |  |  |  |
| Bank of Mont | 6.900 | Oct 16/01 | 104.16 | 5.51 |
| Cdn Imp Bank | 4.500 | Dec 06/99 | 98.73 | 5.41 |
| Imperial Oil | 9.875 | Dec 15/99 | 106.12 | 5.53 |
| Loblaws Co. | 6.650 | Nov 08/27 | 107.69 | 6.08 |
| Royal Bank | 11.000 | Jan 11/02 | 117.46 | 5.53 |
| Union Gas | 8.650 | Nov 10/25 | 133.29 | 6.13 |
| Source: The Financial Post, June 17, 1998, p.45. Used with permission. |  |  |  |  |

Inflation and Returns
■ Key issues:

- What is the difference between a real and a nominal return?
- How can we convert from one to the other?
- Example:

Suppose we have $\$ 1,000$, and Diet Coke costs $\$ 2.00$ per six pack. We can buy 500 six packs. Now suppose the rate of inflation is $5 \%$, so that the price rises to $\$ 2.10$ in one year. We invest the $\$ 1,000$ and it grows to $\$ 1,100$ in one year. What's our return in dollars? In six packs?

Inflation and Returns (continued)

- A. Dollars. Our return is
$(\$ 1100-\$ 1000) / \$ 1000=\$ 100 / \$ 1000=$ $\qquad$ .
( $\therefore$ ) The percentage increase in the amount of green stuff is $10 \%$; our return is $10 \%$.

■ B. Six packs. We can buy $\$ 1100 / \$ 2.10=$ $\qquad$ six packs, so our return is
$(523.81-500) / 500=23.81 / 500=4.76 \%$
( $\therefore$ ) The percentage increase in the amount of brown stuff is 4.76\%; our return is 4.76\%.

Inflation and Returns (continued)

■ Real versus nominal returns:

Your nominal return is the percentage change in the amount of money you have.

Your real return is the percentage change in the amount of stuff you can actually buy.

Inflation and Returns (concluded)

- The relationship between real and nominal returns is described by the Fisher Effect. Let:

| $R$ | $=$ | the nominal return |
| :--- | :--- | :--- |
| $r$ | $=$ | the real return |
| $h$ | $=$ | the inflation rate |

- According to the Fisher Effect:

$$
1+R=(1+r) \times(1+h)
$$

- From the example, the real return is 4.76\%; the nominal return is $10 \%$, and the inflation rate is $5 \%$ :

$$
\begin{gathered}
(1+R)=1.10 \\
(1+r) \times(1+h)=1.0476 \times 1.05=1.10
\end{gathered}
$$

Factors Affecting Bond Yields

Key Issue:
What factors affect observed bond yields?

- Real rate of interest
- Expected future inflation
- Interest rate risk

■ Default risk premium
■ Taxability premium

- Liquidity premium

Quick Quiz

1. Under what conditions will the coupon rate, current yield, and yield-to-maturity be the same?
2. What does it mean when someone says a bond is selling "at par"? At "a discount"? At "a premium"?
3. What is a "transparent" market? Why is transparency important?
4. What is the "Fisher Effect"?
5. What is meant by the "term structure" of interest rates? How is the term structure of interest rates related to the yield curve?

Solution to Problem

■ Reznik Corporation has bonds on the market with 10.5 years to maturity, a yield-to-maturity of 8 percent, and a current price of $\$ 860$. Coupon payments are semiannual. What must the coupon rate be on the bonds?

Total number of coupon payments $=10.5 \times \times 2=21$
Yield-to-maturity per period = 8\%/2=4\%
Maturity value $=F=\$ 1,000$

Solution to Problem (concluded)

- Substituting the known values into the bond pricing equation:

$$
\begin{aligned}
& \begin{array}{l}
\text { Bond } \\
\text { Value }
\end{array}=C \times\left[1-1 /(1+r)^{t}\right] / r+F /(1+r)^{t} \\
& \$ 860=C \times\left[1-1 /(1+.04)^{21}\right] / .04+\$ 1,000 /(1.04)^{21} \\
& C
\end{aligned}=\$ 34.65
$$

So the annual coupon must be $\$ 34.65 \times 2=\$ 69.30$ and the coupon rate is $\$ 69.30 / \$ 1,000 \bar{x} .0693(6.93 \%)$

Solution to Problem 7.13

■ Locate the Government of Canada issue in Figure 7.3 maturing on 1 FEB 04.

- Is this a note or a bond?
- What is its coupon rate?

What is its bid price?

Solution to Problem 7.13 (concluded)

■ Locate the CANADA issue in Fig. 7.3 maturing in 1 FEB 04.

- Is this a note or a bond?

■ It's a bond, since there is no " $n$ " following the maturity year.
■ What is its coupon rate?
■ The coupon rate is $101 / 4$ percent.
■ What is its bid price?
■ The bid price is $\mathbf{1 2 3 . 9 5}$ percent of par.

Solution to Problem

- Bond J has a 4\% coupon and Bond K a 10\% coupon. Both have 10 years to maturity, make semiannual payments, and have $9 \%$ YTMs. If market rates rise by $2 \%$, what is the percentage price change of these bonds? If rates fall by $\mathbf{2 \%}$ ? What does this say about the risk of lower-coupon bonds?


## Current Prices:

## Bond J:

```
PV = $20 [ [1-1/(1.045) 20]/.045 + $1,000/(1.045) 20
    = $
```

$\qquad$

## Bond K:

```
PV = $50\times[1-1/(1.045)20]/.045 + $1,000/(1.045)}\mp@subsup{}{}{20
    = $1065.04
```

Solution to Problem (continued)

Prices if market rates rise by 2\%:
Bond J:

$$
\begin{aligned}
\text { PV } & =\$ 20 \times\left[1-1 /(1.055)^{20}\right] / .055+\$ 1,000 /(1.055)^{20} \\
& =\$ 581.74
\end{aligned}
$$

Bond K:

$=\$$
$\$$

Solution to Problem (continued)

Prices if market rates fall by 2\%:
Bond J:


```
    = $786.82
```

Bond K:
$P V=\$ 50 \times\left[1-1 /(1.035)^{20}\right] / .035+\$ 1,000 /(1.035)^{20}$ $=\$ 1213.19$

Solution to Problem (concluded)

- Percentage Changes in Bond Prices

Bond Prices and Market Rates

|  | 7\% | 9\% | 11\% |
| :---: | :---: | :---: | :---: |
| Bond J | \$786.81 | \$674.80 | \$581.74 |
| \% chg. | (+16.60\%) |  | (__\%) |
| Bond K | \$1,213.19 | \$1,065.04 | \$940.25 |
| \% chg. | (__\%) |  | (-11.72\%) |

The results above demonstrate that, all else equal, the price of the lower-coupon bond changes more (in percentage terms) than the price of the higher-coupon bond when market rates change.

