Lecture 2
Time Value of Money

FINA 614

Basic Definitions

- Present Value – earlier money on a time line
- Future Value – later money on a time line
- Interest rate – "exchange rate" between earlier money and later money
  - Discount rate
  - Cost of capital
  - Opportunity cost of capital
  - Required return

Future Value: General Formula

- $FV = PV(1 + r)^t$
  - FV = future value
  - PV = present value
  - r = period interest rate, expressed as a decimal
  - T = number of periods
- Future value interest factor = $(1 + r)^t$

Future Value – Example 1 – 5.1

- Suppose you invest $1000 for one year at 5% per year. What is the future value in one year?
  - Interest = 1000(.05) = 50
  - Value in one year = principal + interest = 1000 + 50 = 1050
  - Future Value (FV) = 1000(1 + .05) = 1050
- Suppose you leave the money in for another year. How much will you have two years from now?
  - FV = 1000(1.05)(1.05) = $1000(1.05)^2$ = 1102.50

Effects of Compounding

- Simple interest – earn interest on principal only
- Compound interest – earn interest on principal and reinvested interest
- Consider the previous example
  - FV with simple interest = 1000 + 50 + 50 = 1100
  - FV with compound interest = 1102.50
  - The extra 2.50 comes from the interest of .05(50) = 2.50 earned on the first interest payment

Calculator Keys

- Texas Instruments BA-II Plus
  - FV = future value
  - PV = present value
  - I/Y = period interest rate
    - P/Y must equal 1 for the I/Y to be the period rate
    - Interest is entered as a percent, not a decimal
  - N = number of periods
  - Remember to clear the registers (CLR TVM) after each problem
  - Other calculators are similar in format

Small Calculation

- Your objective is to retire at the age of 60. The amount of money that you expect would fulfil your dreams at this age is $500,000. When you are 50 what amount of money you should invest in a risk free account at $4.5% to achieve your goal?

Present Values

- How much do I have to invest today to have some specified amount in the future?
  - $FV = PV(1 + r)^t$
  - Rearrange to solve for $PV = FV / (1 + r)^t$
- When we talk about discounting, we mean finding the present value of some future amount.
- When we talk about the “value” of something, we are talking about the present value unless we specifically indicate that we want the future value.

Important relationships

- For a given interest rate – the longer the time period, the lower the present value
  - Give an example

- For a given time period – the higher the interest rate, the smaller the present value
  - Give an example

Finding the Number of Periods

- Start with basic equation and solve for t (remember your logs)
  - $FV = PV(1 + r)^t$
  - $t = \ln(FV / PV) / \ln(1 + r)$
- You can use the financial keys on the calculator as well, just remember the sign convention.

Recap of equations

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I. Symbols:

PV = Present value, what future cash flows are worth today

FV_t = Future value, what cash flows are worth in the future

r = Interest rate, rate of return, or discount rate per period—typically, but always, one year

t = Number of periods—typically, but not always, the number of years

C = Cash amount

II. Future value of *C* invested at *r* percent for *t* periods:

$FV_t = C \times (1 + r)^t$

The term $(1 + r)^t$ is called the *future value factor.*

III. Present value of *C* to be received in *t* periods at *r* percent per period

$PV = C/(1 + r)^t$

The term $1/(1 + r)^t$ is called the *present value factor.*

IV. The basic present value equation giving the relationship between pre and future value is:

$PV = FV_t/(1 + r)^t$

Another Calculation

You decide to contribute another few years to your retirement fund. You decided to upgrade your life style after 60. Every year, you will put another 10,000 in your fund at the same rate. What would be your fund value at 60?

Decisions, Decisions

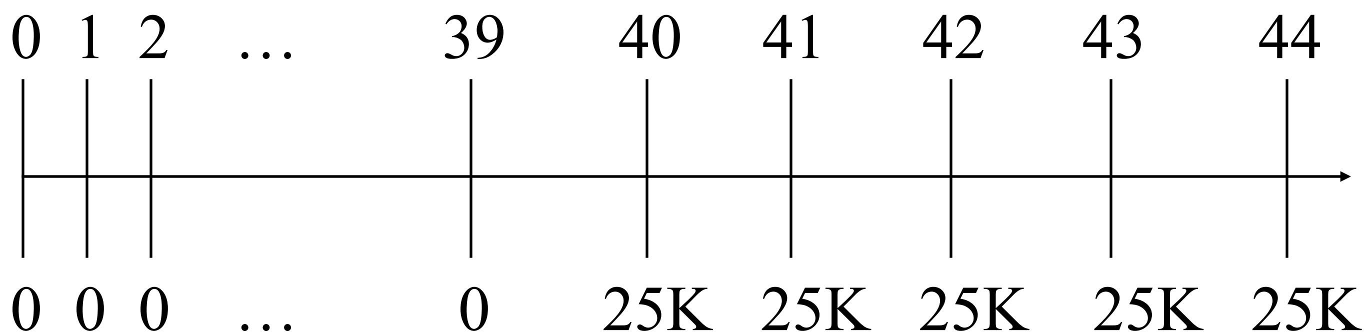
Your broker calls you and tells you that he has this great investment opportunity. If you invest $100 today, you will receive $40 in one year and $75 in two years. If you require a 15% return on investments of this risk, should you take the investment?

Use a time line and calculate manually

Using the financial calculator

- Use the CF keys to compute the value of the investment
  - CF; CF_0 = 0; C01 = 40; F01 = 1; C02 = 75; F02 = 1
  - NPV; I = 15; CPT NPV = 91.49
- No – the broker is charging more than you would be willing to pay.

Saving For Retirement Timeline



Notice that the year 0 cash flow = 0 ($CF_0 = 0$)

The cash flows years 1 – 39 are 0 (C01 = 0; F01 = 39)

The cash flows years 40 – 44 are 25,000 (C02 = 25,000; F02 = 5)

Saving For Retirement continued

- Calculator Approach
  - Use cash flow keys:
    - CF; CF_0 = 0; C01 = 0; F01 = 39; C02 = 25000; F02 = 5; NPV; I = 12; CPT NPV = 1084.71

Annuities and Perpetuities

- Annuity – finite series of equal payments that occur at regular intervals
  - If the first payment occurs at the end of the period, it is called an ordinary annuity
  - If the first payment occurs at the beginning of the period, it is called an annuity due
- Perpetuity – infinite series of equal payments

Annuities and Perpetuities – Basic Formulas

- Perpetuity: PV = C / r
- Annuities:

$$PV = C\left[\frac{1-\frac{1}{(1+r)^t}}{r}\right]$$

$$FV = C\left[\frac{(1+r)^t - 1}{r}\right]$$

Annuity – Example 1

- After carefully going over your budget, you have determined that you can afford to pay $632 per month towards a new sports car. Your bank will lend to you at 1% per month for 48 months. How much can you borrow?

Annuity – Example 1 continued

- You borrow money TODAY so you need to compute the present value.
- Formula Approach

$$PV = 632\left[\frac{1-\frac{1}{(1.01)^{48}}}{.01}\right] = 23{,}999.54$$

- Calculator Approach
  – 48 N; 1 I/Y; -632 PMT; CPT PV = 23,999.54 ($24,000)

Finding the Number of Payments – Example 1

- You ran a little short on your February vacation, so you put $1,000 on your credit card. You can only afford to make the minimum payment of $20 per month. The interest rate on the credit card is 1.5% per month. How long will you need to pay off the $1,000?

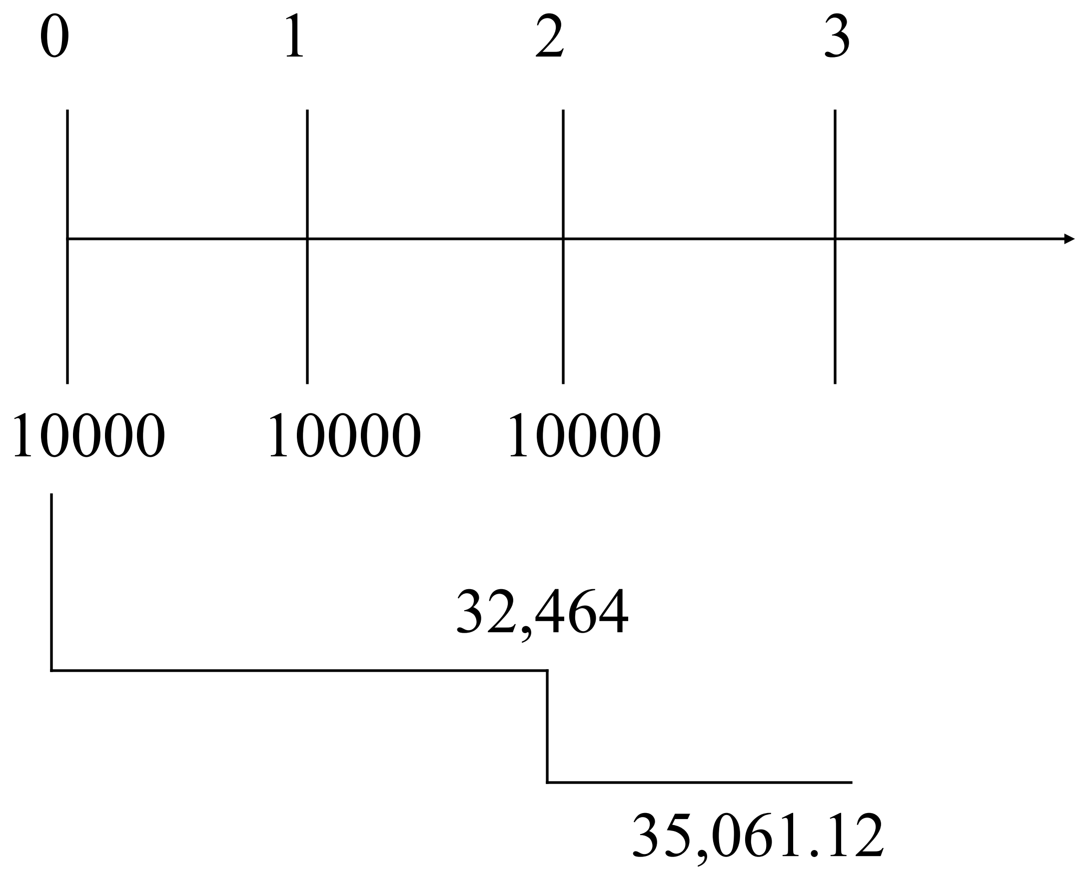
Finding the Number of Payments

- Formula Approach
  - Start with the equation and remember your logs.
    - $1000 = 20(1 - 1/1.015^t) / .015$
    - $.75 = 1 - 1 / 1.015^t$
    - $1 / 1.015^t = .25$
    - $1 / .25 = 1.015^t$
    - $t = \ln(1/.25) / \ln(1.015) = 93.111$ months = 7.76 years
- Calculator Approach
  - The sign convention matters!!!
    - 1.5 I/Y
    - 1000 PV
    - -20 PMT
    - CPT N = 93.111 MONTHS = 7.76 years
- And this is only if you don't charge anything more on the card!

Annuity – Finding the Rate Without a Financial Calculator

- Trial and Error Process
  - Choose an interest rate and compute the PV of the payments based on this rate
  - Compare the computed PV with the actual loan amount
  - If the computed PV > loan amount, then the interest rate is too low
  - If the computed PV < loan amount, then the interest rate is too high
  - Adjust the rate and repeat the process until the computed PV and the loan amount are equal

Annuity Due



Annuity Due

- Formula Approach
  - $FV = 10{,}000[(1.08^3 - 1) / .08](1.08) = 35{,}061.12$
- Calculator Approach
  - 2nd BGN 2nd Set (you should see BGN in the display)
  - 3 N
  - -10,000 PMT
  - 8 I/Y
  - CPT FV = 35,061.12
  - 2nd BGN 2nd Set (be sure to change it back to an ordinary annuity)

Perpetuity

- The Home Bank of Canada want to sell preferred stock at $100 per share. A very similar issue of preferred stock already outstanding has a price of $40 per share and offers a dividend of $1 every quarter. What dividend would the Home Bank have to offer if its preferred stock is going to sell?

Perpetuity – Example 1 continued

- Perpetuity formula: PV = C / r
  - First, find the required return for the comparable issue:
    - 40 = 1 / r
    - r = .025 or 2.5% per quarter
  - Then, using the required return found above, find the dividend for new preferred issue:
    - 100 = C / .025
    - C = 2.50 per quarter

Growing Perpetuity

- The perpetuities discussed so far are annuities with constant payments
- Growing perpetuities have cash flows that grow at a constant rate and continue forever
- Growing perpetuity formula:

$$PV = \frac{C_1}{r - g}$$

Growing Perpetuity

- Hoffstein Corporation is expected to pay a dividend of $3 per share next year. Investors anticipate that the annual dividend will rise by 6% per year forever. The required rate of return is 11%. What is the price of the stock today?

$$PV = \frac{\$3.00}{0.11 - 0.06} = \$60.00$$

Growing Annuity

- Growing annuities have a finite number of growing cash flows
- Growing annuity formula:

$$PV = \frac{C_1}{r-g}\left[1-\left(\frac{1+g}{1+r}\right)^T\right]$$

Growing Annuity

- Gilles Lebouder has just been offered a job at \$50,000 a year. He anticipates his salary will increase by 5% a year until his retirement in 40 years. Given an interest rate of 8%, what is the present value of his lifetime salary?

$$PV = \frac{\$50{,}000}{0.08 - 0.05}\left[1 - \left(\frac{1.05}{1.08}\right)^{40}\right] = \$1{,}126{,}571$$

Recap

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I. Symbols:

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t = Number of periods—typically, but not always, the number of years

C = Cash amount

II. Future value of *C* per period for *t* periods at *r* percent per period:

$FV_t = C \times \{[(1 + r)^t - 1]/r\}$

A series of identical cash flows is called an *annuity,* and the term $[(1 + r)^t - 1]$ called the *annuity future value factor.*

III. Present value of *C* per period for *t* periods at *r* percent per period:

$PV = C \times \{1 - [1/(1 + r)^t]\}/r$

The term $\{1 - [1/(1 + r)^t]\}/r$ is called the *annuity present value factor.*

IV. Present value of a perpetuity of *C* per period:

$PV = C/r$

A *perpetuity* has the same cash flow every year forever.

Annual Percentage Rate

- This is the annual rate that is quoted by law
- By definition APR = period rate times the number of periods per year
- Consequently, to get the period rate we rearrange the APR equation:
  - Period rate = APR / number of periods per year
- You should NEVER divide the effective rate by the number of periods per year – it will NOT give you the period rate

EAR - Formula

$$EAR = \left[1 + \frac{APR}{m}\right]^{m} - 1$$

Remember that the APR is the quoted rate

m is the number of times the interest is compounded in a year

Continuous Compounding

- Sometimes investments or loans are calculated based on continuous compounding
- EAR = $e^q - 1$
  - The e is a special function on the calculator normally denoted by e^x
- Example: What is the effective annual rate of 7% compounded continuously?
  - EAR = $e^{.07} - 1$ = .0725 or 7.25%