Concordia University
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Decision Sciences and Management Information Systems

MBA 608 Statistical Models for Business Decisions

Fall 2010
Section A

Class Test 2

Name: Key

I.D.: ____________________

DIRECTIONS:
The exam is divided into 2 parts: PART A) True of False and Multiple Choice questions, and PART B) Problems to solve. For part A, indicate the most appropriate answer. For part B, work problems in the space provided and please show all work.

Question	Maximum	Your Grade
Part A	35	
Part B		
1	9	
2	8	
3	11	
4	12	
Total:	75	

PART A

True-False (1 point each): Circle the most appropriate response T=True, F=False for the following statements.

1	The t-distribution allows the calculation of confidence intervals for means when the actual standard error is not known.	**T**	**F**
2	A sampling distribution is defined as the probability distribution of possible sample sizes that can be observed from a given population.	**T**	**F**
3	The Central Limit Theorem ensures that the sampling distribution of the sample mean approaches normal as the sample size increases	**T**	**F**
4	An unbiased estimator will have a value, on average across samples, equal to the population values.	**T**	**F**
5	As the confidence level for a confidence interval increases, the width of the interval increases.	**T**	**F**

Multiple choice (2 points each) Circle the most appropriate response

6. For a given α, as we increase the sample size:

   a. The probability of committing a Type II error and the power of the test decrease
   b. The probability of committing a Type II error increases
   c. The probability of committing a Type II error decreases and the power of the test increases
   d. There is no impact on the probability of a Type II error.
   e. None of the above

7. When trying to test $H_0: \mu \leq 10$ against $H_A: \mu > 10$, Harry Arms calculated a Z-value of -1.79. For this test, Harry's p-value is

   a. 0.0367
   b. 0.9633
   c. 0.4633
   d. 0.5367

8. To test $H_0: \mu = 50$ against $H_1: \mu \neq 50$, a statistician obtained a random sample with the following characteristics: n=11, $\overline{X} = 60$, S=14. The p-value for this test would be

a. Between 0.02 and 0.05
b. Between 0.01 and 0.025
c. Between 0.025 and 0.05
d. Between 0.05 and 0.10
e. 0.0089

9. When n=81, $\overline{X} = 9.6$, and σ^2=0.81 with 1-α=0.95, then the lower confidence limit is

a. 9.796
b. 9.424
c. 9.404
d. -9.404
e. not sufficient information

10. To reduce the width of a confidence interval for a mean by one-half, when the original sample was of size 36, how many additional sample items are needed?

a. 72
b. 144
c. 108
d. cannot be determined from the given information

A real estate company is interested in testing whether, on average, families in City A have been living in the current homes for less time than families in City B have. A random sample of 100 families from City A and a random sample of 150 families from City B yield the following data on length of residence time in current homes.

City A:	$\overline{X} = 35$ months	$S^2 = 900$
City B:	$\overline{X} = 50$ months	$S^2=1050$

11. Suppose μ_A and μ_B represent the true average amount of time (in months) families have lived in City A and City B, respectively. Which of the following represents the relevant hypotheses tested by the real estate company?

a. $H_0: \mu_A > \mu_B$
$H_1: \mu_A \neq \mu_B$

b $H_0: \mu_A = \mu_B$
$H_1: \mu_A \neq \mu_B$

c. $H_0: \mu_A \geq \mu_B$
$H_1: \mu_A < \mu_B$

d. $H_0: \mu_A \leq \mu_B$
$H_1: \mu_A > \mu_B$

12. With which of the following significance levels will we most frequently reject the null hypothesis?

a. 0.10
b. 0.005
c. 0.05
d. 0.01

13. For a given level of significance, if the sample size is increased, the power of the test will

a. increase.
b. decrease.
c. remain the same.
d. be undeterminable.

14. In a statistical test, the probability of obtaining a result equal to or even more extreme than the value observed – given the null hypothesis is true—is known as

a. a Type I error.
b. statistical power.
c. homoscedasticity.
d. the P value.

15. If the P value is less than α in a two-tailed test

a. the null hypothesis should not be rejected.
b. the null hypothesis should be rejected.
c. a one-tailed test should be used.
d. α should be changed.

16. A sample of size 25 provided a sample variance of 400. The sample standard error, which in this case is equal to 4, is best described as:

a. the standard deviation of the means calculated from samples of size 25.
b. one-half of the width of a symmetric interval about the mean containing 90% of all observations.
c. the estimate of the standard deviation of means calculated from samples of size 25
d. the estimated standard deviation for individuals from this population.

17. For a normal distribution, approximately 68% of all observations lie within on standard deviation of the mean. What percentage of sample means (calculated from samples of size 4) would not be contained in the same interval?

a. 7.56%
b. 4.56%
c. .26%
d. less than .1%

18. A Type I error is committed when

a. we reject a null hypothesis that is true.
b. we don't reject a null hypothesis that is true.
c. we reject a null hypothesis that is false.
d. we don't reject a null hypothesis that is false.

19. The t test for the difference between the means of two independent populations assumes that the respective

a. sample sizes are equal.
b. sample variances are equal.
c. populations are approximately normal.
d. b. and c.
e. All of the above.

20. Given that the null hypothesis is true,

a. the probability of a type II error is 1-a
b. you cannot make a type II error
c. the probability of a type I error is 1.00
d. the probability of a type II error is equal to a

PART B

Question 1 (9 points)

a) The average tuition at a 4-year private college is $14,858 with a standard deviation of $2,450. A researcher has a theory that the average tuition at a 4-year private college is actually higher than that, and she sets out to demonstrate that the theory is true by randomly sampling 40 private colleges. Suppose the sample of private colleges produces a sample average tuition of $15,607. Use $\alpha = 0.10$ to test the researcher theory. (5 points)

① $H_0: \mu \le 14{,}858$

$H_A: \mu > 14{,}858$

② $\alpha = .1$

C.V. = 1.28

③ $z = \dfrac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \dfrac{15607 - 14858}{2450/\sqrt{40}} \quad \dfrac{749}{2450/6.32} = \dfrac{749}{387.66} = 1.93$

④ $1.93 > 1.28$ Rej. H_0

b). A bank officer wants to determine the amount of the average total monthly deposits per customer at the bank. He believes an estimate of this average amount using a confidence interval is sufficient. How large should he take to be within $200 of the actual average with 99% confidence? He assumes the standard deviation of total monthly deposits for all customers is about $1,000. (4 points)

$n = (z)^2 \sigma^2 / (E)^2 = (2.575)^2 (1000)^2 / (200)^2 = 165.77$

$= 166$

Question 2 (8 points)

a) A regional survey of 560 companies asked the vice president of operations how satisfied he or she was with the software support being received from the computer staff of the company. Suppose 33% of the 560 vice presidents said they were satisfied. Construct a 99% confidence interval for the proportion of vice presidents who would have said they were satisfied with the software support if a census had been taken. (4 points)

$P(\bar{p} - z\sigma_p \leq p \leq \bar{p} + z\sigma_{\bar{x}}) = 1-\alpha$

$P\left(.33 - 2.575\sqrt{\frac{(.33)(.67)}{560}} \leq p \leq .33 + 2.575\sqrt{\frac{(.33)(.67)}{560}}\right)$

$\sqrt{.0003948}$

$= .0198695$

$\times 2.575$

$.05116$

$P(.33 - .05 \leq p \leq .33 + .05) = .99$

$P(.28 \leq p \leq .38) = .99$

b) What proportion of secretaries of Fortune 500 companies has a personal computer at his or her work station? You want to answer this question by conducting a random survey. How large a sample should you take if you want to be 95% confident of the results and you want the width of the confidence interval to be no more than .10? Assume no one has any idea of what the proportion actually is. (4 points)

$1.96 = .05 / \sqrt{\frac{(.5)(.5)}{N}}$

$(1.96)^2 = (.05)^2 / .25/N$

$(1.96)^2 (.25) / (.05)^2$

$= 3.8416(.25) / .0025$

$= 384.16$

$\simeq 385$

Question 3 (11 points)

A Canadian Automobile Association (CAA) study investigated the question of whether a man or a woman was more likely to stop and ask for directions. The situation referred to in the study stated the following: "If you and your spouse are driving together and become lost, would you stop and ask for directions?" A sample representative of the data used by CAA showed 300 of 811 women would stop and ask and ask for directions, while 255 of 750 men said that they would stop and ask for directions.

a. Test the appropriate hypothesis to determine if it is likely that women would be more likely than men to say that they would stop and ask for directions (use a .05 level of significance)

① $H_0: \pi_1 \le \pi_2$

$H_1: \pi_1 > \pi_2$

$\bar{p} = \dfrac{X_1 + X_2}{n_1 + n_2} = \dfrac{300+255}{811+750} = .3555$

② $\alpha = 1.645$

③ $Z = \dfrac{\bar{P}_1 - \bar{P}_2}{\sqrt{\bar{p}(1-\bar{p})\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = \dfrac{.3699 - .3400}{\sqrt{.3555(1-.3555)\left(\frac{1}{811}+\frac{1}{750}\right)}}$

$= .0299 / .02426$

$= 1.23$

④ $1.23 < 1.645$ ∴ Do Not Rej

b. Calculate the p-value for the hypothesis test in part "a" above, and interpret its meaning

P-value $= P(Z > 1.23) = 1 - .8907 = .1093$

$P(Z > 1.24) = .1075$

C. Construct a 90% confidence interval estimate for the differences between population proportions of men and women who said they would stop and ask for directions, and interpret the meaning of the confidence interval.

$$P_1 - P_2 \pm Z_{\alpha/2} \sqrt{\frac{P_1(1-P_1)}{n_1} + \frac{P_2(1-P_2)}{n_2}}$$

$$\left(\frac{300}{811} - \frac{255}{750}\right) \pm 1.645 \sqrt{\frac{.3699 \times .6301}{811} + \frac{(.34)(.66)}{750}}$$

$$.02991 \pm 1.645 \sqrt{.00029 + .00030}$$

$$.02991 \pm 1.645(.02427)$$

$$P(-.01006 \le \pi_1 - \pi_2 \le .06986) = .90$$

Question 4 (12 points)

An auditor for a government agency is assigned to the task of evaluating reimbursement for office visits to doctors paid by Medicare. The audit is to be conducted for all Medicare payments in a particular area during a certain month. An audit is conducted on a sample of 60 of the reimbursements with the following results:

Average amount of reimbursement: $96.65
Standard deviation of reimbursement: $35.75
Number of reimbursements where an incorrect amount was provided: 12

a. At the 0.05 level of significance, is there evidence that the proportion of incorrect reimbursements in the population is greater than 10%? What is the p-value of the test? (5 points)

$\bar{p} = 12/60 = .2$

P-value $= P(Z > 2.58 \mid p = .10) =$
$1 - .9951 = .0049$

① $H_0 : \pi \leq .10$
$H_1 : \pi > .10$

② $\alpha = .05$
C.V $= 1.645$

③ $Z = \dfrac{p - \pi}{\sigma_p} = \dfrac{.2 - .1}{\sqrt{\dfrac{(.1)(.9)}{60}}}$

$= \dfrac{.1}{\sqrt{\dfrac{.09}{60}}} = \dfrac{.1}{\sqrt{.0015}} = \dfrac{.1}{.0387}$

$= 2.58$

④ $2.58 > 1.645$ ∴ Reject H_0

⑤ BASED Upon the CRiteria established in step 2, we do NOT believe that this sample came from a population WITH A proportion equal to .10
The proportion of incorrect reimbursments is probably more than .10

b. At the 0.05 level of significance, is there evidence that the average reimbursement is less than \$100.00? What is the p-value of the test? (5 points)

① $H_0: \mu \geq 100$
$H_1: \mu < 100$
② $\alpha = .05$
$C.V. = t_{(59, .05)} = -1.6711$
③ $t = \bar{x} - \mu / \sigma_{\bar{x}} = 96.65 - 100 / 35.75/\sqrt{60}$
$= -3.35 / 35.75/7.75 = 3.35/4.67$
$= -.717$

④ $-.717 > -1.6711$ ∴ Do Not Rej
⑤ BASED upon...

P-VALUE = $P(t < -.7171 \mid df = 59)$
= between .1 and .25

c. If the level of significance in part a. and part b. was changed to 0.10, would it change the decision change? Justify. (2 points)

a) C.V. = +1.28 No 2.58 > 1.28 Still reject

b) C.V. = -1.2961 No -.66 > -1.2961 Still Do Not Rej