# Risk management

**12 JAN**

Software to get ($50 for student):

Decision Tools Suite

* Palisade
* @Risk
* Precision Trees

<http://www.palisade.com/decisiontools_suite/>

Book: Risk Management & Financial Investment

*By: John Hall*

## CH7: Valuation and Scenario Analysis

Valuation: Find average CF in the future and PV of CFs

Scenario Analysis: Attempt to define possible outcomes with focus on extreme outcomes

Call Option

Strike price: k

Value: SP – k; floors at 0$

2 kinds: European: can only exercise at a given time; American; can exercise any time after 3 months

If have a call option on a non-dividend paying stock; it will never be in your interest to exercise it before maturity. When holding, can do 3 things:

* Sell; collect SP-k
* Hold on
* Sell it on the market to collect risk premium (MV always > than SP-k)

Therefore, in terms of valuation, for practical purposes, valuation of European option = American option (premium for flexibility has a value of 0$).

Only applies to non-dividend call (no dividend, no puts); everywhere else, value of European is always < American.

**Example:**

*Valuation (Considering Average CFs):*

A Company sells 1,000,000 1y European call options on a stock.

Current SP: $50

Strike k: $55

Valuation of options: $4.5M

If options are sold for $5M; profit = 0.5M$

*Scenario Analysis (considering extreme CFs):*

5% chance that stock price exceeds 80$ in 1 year -> loss= ($80-$55)\*1M - $5M = $20M

## Volatility and Asset Prices

In a small period of time Δt

Sigma σ = volatility

E = N(0,1)

ΔS/S = actual + error = stock price

ΔS/S = u Δt + σ \* sqrt(T) E

U = investment average return

Σ = volatility

It can be shown that in statistics, log(stock price is standardly distributed)

Normal distribution has N(u, σ^2)

Ln(ST) ~ φ[ln S0 + (u- σ^2/2)T, σ^2 \* T]

Stock prices are log normally distributed:



Stock prices can’t be negative because of limited liabilities

But stock returns are normally distributed.

**Example:**

What is probability that stock price < certain value at time T

Prob (ST < V) = N [ (lnV – lnS0 – (u- σ^2/2)T) / σ sqrt(T) ] = N (-d2)

Prob (ST < V) = N [ (ln(V/S0) – (u - σ^2/2)T) / σ sqrt(T) ]

Area under the curve

Might also be interested in:

prob (St > V) = 1 – N (-d2) = N(d2)

where: d2 = ln ( S0/V) + (u - σ^2/2)T

*Note: No need to memorize theory; just understand it*

**Example:**

Want to find V such that:

P(ST > V) = q; for example, if q=0.1, find V

P(ST > V) = N(d2) = q, where d2= N-1(q)

d2 = [ln(S0/V + (u- σ2/2)T] / σ sqrt(T)

Solve for V:

V = S0 exp[(u- σ2/2)T – N-1(q) \* σ sqrt(T)]

Similarly, if we are looking for V such that prob(ST < V) = q

V = S0 exp[(u- σ2/2)T + N-1(q) \* σ sqrt(T)]

## Risk Neutral Valuation

**Example:**

S0 value is $50; in a given period of time, can either increase or decrease by 20%, making

S1=$40 or $60, where P(S1=$40) = 0.7 and P(S1=$40) = 0.3

Call option with k=$55 will be worth $5 or $0; risk free rate r = 10%; goal: find value of option

Method 1:

Value = [($5 \* 0,7) + ($0 \* 0,3)] / (1 + R)

Where R = r + risk premium

Problem is that need to find risk premium

Method 2: Create a replicating portfolio to mimic behavior of call option

Buy Δshares and borrow B = > Gives 2 formulas with 2 unknowns

If S1 = $60 => Δ$60 + B \* 1.1 = $5

If S1 = $40 => Δ$40 + B \* 1.1 = $0

Δ$20 = $5 => Δ = ¼ making B = -10/11 = -9.09

Value of option: ¼ \* $50 – $9.09 = $3.41

\*probabilities do not really matter, since assumption is that market pricing of stock already captures probabilities in pricing. If probability changes, price of stock will also change.

This is Black Sholes model using discrete time in a complete market (where all payoff can be achieved by any instruments: derivatives are redundant securities, therefore call options are the same as shares)

Other type of question

Stock Price is $50, what should the probabilities (p and [1-p]) be such that the expected growth rate of the stock = Rf

$50 \* (1 + 0.1) = $60p + $40 ( 1-p)

55 = 20p + 40

P = 0.75

Option payout:

[($5 \* 0.75) + ($0 \* 0.25)] / (1 + 0.1) = 3.41

If using risk neutral probabilities, stock always grows at risk free rate. Advantage: Can simply discount at risk free rate; don’t need to worry about risk premium.

Actual growth rate of the stock u and volatility σ, but since risk free, can simply replace u by Rf

Summary:

* Prices using synthetic portfolio
* Need to be equal, otherwise provides arbitrage opportunities
* Performed some transformations, and arrived at the conclusion that stock price grows at Rf rate
* Only happens when you don’t care about the price of the stock, why it is called risk neutral pricing