FINA 695 Assignment 1

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Answer the following questions. Show your work.
Due in the class on March 29. (postponed 1 week)
You are expected to do the assignment on your own. Please do not take help from others.

1. (a) The current stock price is \$50, the (instantaneous) expected growth rate is 12%, and the volatility is 30%. What is the probability that the stock price will be greater than \$90 after 2 years? (5 marks)

S_0 = \$50	$P(S_t > V) = N[(\ln(S_0/V) + (\mu-\sigma^2/2)T \;/\; \sigma\sqrt{T}]$
μ = 12%	$P(S_2 > \$90) = N[(\ln(\$50/\$90) + (.12-.3^2/2)*2 \;/\; 0.3\sqrt{2}]$
σ=30%	$P(S_2 > \$90) = N[(-0.5878 + .15) / .4243]$
T=2	$P(S_2 > \$90) = N[-1.03187] = 0.151$
V = \$90	
$P(S_2 > \$90)$ = ?	**$P(S_2 > \$90)$ = 15.1%**

(b) What is the stock price that has a probability of 25% of being exceeded in 18 months?

S_0 = \$50	$V = S_0 \;\exp[(\mu-\sigma^2/2)T - N^{-1}(q)*\sigma\sqrt{T}]$
μ = 12%	$V = \$50 \exp[(0.12-0.3^2/2)1.5 - N^{-1}(0.25)*0.3\sqrt{1.5}]$
σ=30%	$V = \$50 \exp[0.1125 - (-.674) * (0.3674)]$
T=1.5	$V = \$50 \exp[0.36]$
V = ?	$V = \$50 * 1.434$
$P(S_{1.5} > V)$ = 25%	**V = \$71.68**

2. (a) A binary option on the above stock pays \$10 if the stock price is greater than \$55 after 3 months. If the continuously compounded interest rate is 6%, what is the price of this binary option? (5 marks)
   1) *Assume stock grows at Rf & find Risk Neutral $P(S_T>V)$*
   2) *Determine payoff at t=T*
   3) *Discount back at Rf rate to find derivative value*

<u>Probability of return</u>

S_0 = \$50	$P(S_t > V) = N[(\ln(S_0/V) + (Rf-\sigma^2/2)T \;/\; \sigma\sqrt{T}]$
μ = Rf = 6%	$P(S_{1/4} > \$55) = N[(\ln(\$50/\$55) + (0.06-0.3^2/2)/4) \;/\; 0.3\sqrt{0.25}]$
σ=30%	$P(S_{1/4} > \$55) = N[(-0.0953 + 0.00375) \;/\; 0.15]$
T=1/4	$P(S_{1/4} > \$55) = N[-0.6104]$
V = \$55	**$P(S_{1/4} > \$55) = 0.2708$**

<u>EV of Payout @ t=3months</u>

EV = PMT * $P(S_{1/4} > \$55)$
EV = \$10 * 0.2708
EV = \$2.71

<u>Present Value of Payout</u>

PV = FV * exp (-Rf*T)
PV = \$2.71 exp (-0.06*1/4)
PV = \$2.71 * 0.9851
PV = \$2.67

Another option on the above stock pays $10 if after 1 year the stock price is between $60 and $70 and $15 if the stock price is greater than $70. What is the price of this option?

<u>Probability of return</u>

S_0 = $50 — $P(S_t > V) = N[(\ln(S_0/V) + (Rf-\sigma^2/2)T \;/\; \sigma\sqrt{T}]$

μ = Rf = 6% — $P(S_1 > \$70) = N[(\ln(\$50/\$70) + (0.06-.03^2/2)1 \;/\; 0.3\sqrt{1}]$

σ=30% — $P(S_1 > \$70) = N[(-0.33647 + 0.015) /0.3]$

T=1 — $P(S_1 > \$70) = N[-1.07157]$

V = $70 — **$P(S_1 > \$70) = 0.14196$**

$P(S_1 > \$70)$

S_0 = $50

μ = Rf = 6% — $P(S_1 > \$60) = N[(\ln(\$50/\$60) + (0.06-.03^2/2)1 \;/\; 0.3\sqrt{1}]$

σ=30% — $P(S_1 > \$60) = N[(-0.18232 + 0.015) /0.3]$

T=1 — $P(S_1 > \$60) = N[-0.107738]$

V = $60 — **$P(S_1 > \$60) = 0.4571$**

$P(S_1 > \$60)$

$P(\$60 > S_1 > \$70) = P(S_1 > \$60) - P(S_1 > \$70)$

$P(\$60 > S_1 > \$70) = 0.4571 – 0.14196$

$P(\$60 > S_1 > \$70) = 0.3151$

<u>EV of Payout @ t=1yr</u>

$EV = PMT_{70} * P(S_1 > \$70) + PMT_{60-70} * P(\$60 > S_1 > \$70)$

EV = $15 * 0.14196 + $10 * 0.3151

EV = $2.13 + $3.15

EV = $5.28

<u>Present Value of Payout</u>

PV = FV * exp (-Rf*T)

PV = $5.28 exp (-0.06*1)

PV = $5.28 * 0.9418

PV = $4.97

3. (a) Suppose the gain from a portfolio during six months is normally distributed with mean of $5 million and standard deviation of $15 million. What is the VaR of the portfolio at 99% level of confidence?

μ = $5M
σ = $15M
$N^{-1}(99\%) = -2.33$

Let X be the gain of the portfolio,
then X ~ N($5M, $15M)
Y = (X- μ)/ σ
Y = (X – 5M$)/15M$

P(-2.33 < Y) = 99%
P(-2.33 < (X – 5M$)/15M$) = 99%

Solve for
-2.33 < (X – 5M$)/15M$
X > -2.33 * 15M$ + 5M$
X > 29.95M$

VaR with 99% confidence is 29.95M$

(b) Suppose that for a project, all outcomes from a loss of $20 million to gain of $40 million are equally likely. What is the VaR at the 95% confidence level?

(5 marks)

The loss of the project has uniform distribution extending form [-$20M, $40M]

<u>Range</u>
$40M – (-$20M) = $60M
5% of range = 5% of $60M = 3M$

<u>Offset of range % from lower limit of range</u>
-$20M + $3M = -$17M

<u>The VaR at 95% is therefore:</u>

$17M

4. (a) The price of gold at the close of trading yesterday was $312 and its volatility was estimated to be 1.6% per day. The price at the close of trading today is $306. What is the estimate of volatility for tomorrow according to EWMA model with $\lambda = 0.92$? (Note: Volatility is std dev; the square root of Varinace-σ^2) (5 marks)

   $\lambda = 0.92$
   n = tomorrow
   n-1 = today
   n-2 = yesterday

   Historical Volatility: $0.016 = \sqrt{\sigma^2}$
   Historical Variance: $\sigma_{n-1}^2 = 0.016^2 = 0.000256$

   $\mu_{n-1} = (P_{n-1}-P_{n-2}) / P_{n-2}$
   $\mu_{n-1} = (\$312 - \$306) / \$306$
   $\mu_{n-1} = -0.019230769$

   $\sigma_n^2 = \lambda * \sigma_{n-1}^2 + (1-\lambda) * \mu_{n-1}^2$
   $\sigma_n^2 = (0.92) * (0.000256) + (1-0.92) * (-0.019230769)^2$

   $\sigma_n^2 = -0.001302942$

   (b) For the above problem, what is the estimate of volatility for tomorrow according to GARCH(1, 1) model with $\omega = 0.0000027075$, $\alpha = 0.05$, and $\beta = 0.92$?

   GARCH(p, q):
   $\sigma_n^2 = \omega + \alpha * \sum_1^q \mu_{n-q}^2 + \sum_1^p \beta * \sigma_{n-p}^2$

   GARCH(1, 1):
   $\sigma_n^2 = \omega + \alpha * \sum_1^q \mu_{n-q}^2 + \sum_1^p \beta * \sigma_{n-p}^2$
   $\sigma_n^2 = \omega + \alpha * \mu_{n-1}^2 + \beta * \sigma_{n-1}^2$
   $\sigma_n^2 = (0.0000027075) + (0.05) * (-0.019230769)^2 + (0.92) * (0.016)^2$
   $\sigma_n^2 = (0.0000027075) + (0.05) * 0.000369822) + (0.92) * (0.000256)$
   $\sigma_n^2 = (0.0000027075) + (0.0000184911) + (0.00023552)$

   $\sigma_n^2 = 0.0002567186$

   (c) For the above problem, what is the long-run average volatility?
   $V_L = \beta * \sigma_{n-1}^2 \ / \ (1 - \alpha * \mu_{n-1}^2 + \beta * \sigma_{n-1}^2)$
   $V_L = (0.00023552) / (1 – (0.0000184911 + 0.00023552))$

   $V_L = 0.00023558$

5. Go to www.ca.finance.yahoo.com. Download the daily stock prices of Microsoft for the 2-year period from November 09, 2013 to November 09, 2015. To do this, enter the ticker symbol of Microsoft, which is MSFT, and click on Look Up. Then, under QUOTES, click on Historical Prices. Enter the date range and get the daily prices. Download the prices for the past three years. Work with the Adj. close prices to answer the following questions:

   Please note that when you download the data for Microsoft, the data is sorted from the newest to the oldest. You have to first sort the data from the oldest to the newest (November 09, 2013 to November 09, 2015, in that order) before you can estimate the EWMA and GARCH(1, 1) models. (10 marks)

   Estimate the parameters of EWMA and GARCH (1, 1) for Microsoft.

EWMA

Create column for Return: $\mu_{n-1} = (P_{n-1} - P_{n-2}) / P_{n-2}$

Create column for Variance using EWMA: $\sigma_n^2 = \lambda * \sigma_{n-1}^2 + (1-\lambda) * \mu_{n-1}^2$

Create column for prob: $-\ln(\sigma^2) - \mu^2 / \sigma^2$

Set initial $\lambda = 0.5$

Set Likelihood = sum(prob)

Solve to maximize likelihood by tweaking λ with constraint $\lambda < 1$

Lambda	**= 0.975660393**

GARCH(1, 1)

Create column for Return: $\mu_{n-1} = (P_{n-1} - P_{n-2}) / P_{n-2}$

Create column for Variance using GARCH(1,1): $\sigma_n^2 = \omega + \alpha * \mu_{n-1}^2 + \beta * \sigma_{n-1}^2$

Create column for prob: $-\ln(\sigma^2) - \mu^2 / \sigma^2$

Set initial ω *100000 = 0.5, $\alpha = 0.5$, $\beta = 0.5$

Set Likelihood = sum(prob)

Solve for to max(likelihood) by tweaking ω *100000, α & β *0.1

ω = 0.000002970244
α = 0.009245135999
β = 0.980548654662

6. Use the data in the spreadsheet "Data for Problem 6" to answer this question.

Suppose that a portfolio has (in $000S) invested 10,000 in DJIA, 15,000 in FTSE, 10,000 in CAC 40 and 15,000 in Nikkei 225.

(a) What is the 1-day 95% VaR using historical simulation?

(See Excel for work)

$257,581

(b) What is the 1-day 95% VaR using the weighting-of-observations? (use λ = 0.95)

(See Excel for work)

$553,300

(c) What is the 1-day 95% VaR using the volatility-updating? Use the EWMA to answer this question. But before you can estimate the VaR, you have to estimate the value of λ for each of the 4 indices.

(See Excel for work)

λ_{DJIA} = 0.970820596

$\lambda_{FTSE\ 100}$ = 0.88877916

$\lambda_{CAC\ 40}$ = 0.912240631

$\lambda_{Nikkei\ 225}$ = 0.908807558

1-day 95% VaR = $1,455,427

(15 marks)

7. Random variable V_1 is uniformly distributed with values between 0 and 2. Random variable V_2 is uniformly distributed with values between -2 and 4. Produce a <u>cumulative joint probability distribution</u> for V_1 and V_2 table using Gaussian copula with a correlation of 0.45.

V_2

V_1	-1.00	1.50	3.00
0.40			
1.20			
1.80			

Note: Using a Guassian copula with a correlation of 0.45 does not imply that the random variable V_1 and V_2 will also have a correlation of 0.45. This is because the transformation between the given uniform random variables and the standard normal variables in non-linear. Generally, the copula correlation is not the correlation of variables combined into a joint distribution using the copula.

(5 marks)

8. The equity value is $6 million and the volatility of equity is 40%. The debt to be repaid in two years is $10 million. The risk-free rate is 5%. Given this information, what is the risk-neutral probability of default by the firm? (Book page 420)

(10 arks)

$E_0 = V_0N(d_1) - De^{-rT} N(d_2)$
$d_1 = (\ln(V_0/D) + (r+\sigma^2/2)T) / \sigma\sqrt{T}$
$d_2 = d_1 - \sigma\sqrt{T}$

E_0 = $6M
σ_E = 40%
Rf = 0.05
T = 2
D = $10M

$d_1 = (\ln(V_0/D) + (r+\sigma^2/2)T) / \sigma\sqrt{T}$
$d_1 * \sigma\sqrt{T} - (r+\sigma^2/2)T = \ln(V_0/D)$

9. The following table has information about four stocks and your investments in these stocks.

	Stock 1	Stock 2	Stock 3	Stock 4
Current stock price	$27	$31	$36	$45
Shares purchased	100	200	200	400
Mean annual growth rate	13%	22%	18%	24%
Annual volatility	16%	25%	21%	30%

Your investment horizon is one year. Assume a constant correlation of 0.65 among the four stocks. Set the number of iterations to 5,000 in your simulation.

a. What is the expected return (simple return) on your investment?

Answer:______________________________

b. What is the probability that you would end up with a negative return?

Answer:______________________________

c. What is the VaR at 90% level of confidence (in dollars)?

Answer:______________________________

d. What is the probability that you will earn a return between 10% and 20%?

Answer:______________________________

e. What is the probability that you will earn a return greater than 30%?

Answer:______________________________
(10 arks)

10. In the spreadsheet named Data on Returns of Financial Assets, you will find data on returns on Treasury bills, Treasury bonds, stocks, and inflation in columns L to P from the year 1946 to 2000. You are planning for your retirement and will be saving $10,000 for the next 40 years. A constant percentage of your money will be invested in Treasury bills, Treasury bonds, stocks.

a. What should these percentages be so that you maximize the 5th percentile of the money you are likely to have (in present value terms) after 40 years? You decide not to invest more than 50% of you money in stocks.

To answer this question, sample from the given historical returns data. Use the dampening factor of 0.95. Set the number of iterations to 500 and the runtime to 3 minutes.

Answer:______________________________

b. What should the weights be if you want to maximize your expected value of your investment (in present value terms) after 40 years? Use the same constraints and setting above.

Answer:______________________________

c. What should the weights be if you want to minimize the probability that the value of your investment (in present value terms) after 40 years is less than $1 million?

Answer:______________________________
(10 arks)

11. The price of a certain stock is $40 and its volatility is 30%. The risk-free rate is 5%.

a. Consider a down-and-out put barrier option with a strike price of $35 and a knock-out barrier of $25. The option maturity is 6 months and assume 260 trading days in a year. What is the price of this option? To determine the price using simulation, set the number of iterations to 5,000.

Answer:______________________________

b. Using the information above about the stock, what is the price of down-and-in put barrier option with the knock-in barrier of $25?

Answer:______________________________

c. Consider an up-and-out call barrier option with a strike price of $45 and the knock-out barrier of $55. What is the price of this option?

Answer:______________________________

d. What is the price of an up-and-in barrier call option with a strike price of $45 and the knock-in barrier of $55?

Answer:______________________________
(10 marks)